Corner Symmetry and Quantum Geometry

角对称性与量子几何

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Abstract

摘要

By virtue of the Noether theorems, the vast gauge redundancy of general relativity provides us with a rich algebra of boundary charges that generate physical symmetries. These charges are located at codimension-2 entangling surfaces called corners. The presence of non-trivial corner symmetries associated with any entangling cut provides stringent constraints on the theory's mathematical structure and a guide through quantization. This report reviews new and recent results for non-perturbative quantum gravity, which are natural consequences of this structure. First, we establish that the corner symmetry derived from the gauge principle encodes quantum entanglement across internal boundaries. We also explain how the quantum representation of the corner symmetry algebra provides us with a notion of quantum geometry. We then focus our discussion on the first-order formulation of gravity and show how many results obtained in the continuum connect naturally with the previous results in loop quantum gravity. In particular, we show that it is possible to get, purely from quantization and without discretization, an area operator with discrete spectrum, which is covariant under local Lorentz symmetry. We emphasize that while loop gravity correctly captures some of the gravitational quantum numbers, it does not capture all of them, which points toward important directions for future developments. Finally, we discuss the understanding of the gravitational dynamics along null surfaces as a conservation of symmetry charges associated with a Carrollian fluid.

借助诺特定理，广义相对论的大量规范冗余为我们提供了生成物理对称性的丰富边界荷代数。这些荷位于被称为角点的余维 2 纠缠曲面上。与任意纠缠切割关联的非平凡角点对称性的存在，为该理论的数学结构给出了严格约束，并为量子化研究提供了指引。本综述梳理了由该结构自然导出的非微扰量子引力最新研究成果。首先，我们论证由规范原理导出的角点对称性可编码内边界之间的量子纠缠。我们还解释了角点对称代数的量子表示如何为我们给出量子几何的概念。随后我们聚焦引力的一阶表述，展示连续背景下得到的诸多结果如何自然联系圈量子引力的已有研究结论。特别地，我们证明仅通过量子化即可在不离散化的前提下得到局域洛伦兹对称性下协变的离散谱面积算符。我们需要强调，尽管圈引力正确捕捉到了部分引力量子数，但它并未囊括全部量子数，这指明了未来发展的重要方向。最后，我们讨论了如何将类光面上的引力动力学理解为与卡罗尔流体关联的对称荷守恒。

Keywords

关键词

Boundary degrees of freedom - Holography . Loop Quantum Gravity . Noether charges - Symmetry algebras - Corner symmetries - Carrollian geometry - Representation theory of infinite-dimensional groups

边界自由度 - 全息原理. 圈量子引力. 诺特荷 - 对称代数 - 边界角对称性 - 卡罗尔几何 - 无限维群表示论

Introduction

引言

Quantum gravity is one of our time's most captivating and challenging theoretical puzzles. Many different top-down approaches have been proposed to address it, such as (This is by no means a complete list. See, for example, [1].) String Theory, AdS/CFT Holography (Celestial holography can also be understood as a bottom-up perspective on quantum gravity that is perturbative in nature.), and Loop Quantum Gravity (LQG hereafter). Despite many successes, these approaches are based on fundamentally different foundational principles and seem irreconcilable. What is missing, in our view, is a bottom-up non-perturbative perspective that provides, from our understanding of semi-classical gravity and its symmetries, solid foundations and tests that any top-down approach to quantum gravity ought to pass. It includes one of the foundational elements of holography, namely the fact that the Hamiltonian of a gravitational system with fixed boundary conditions is entirely contained on its boundary. It also respects one of the fundamental tenants of LQG, namely background independence, and the foundations of the equivalence principle, which implies that gravitational dynamics is the expression of its symmetry. The local holography approach views both holography and symmetry as two sides of the same coin. In this review, we focus on presenting the connection of local holography with LQG and some of its results such as the appearance of quantum geometry, leaving aside a more general presentation of its link with traditional holography.

量子引力是当今最引人入胜、最具挑战性的理论难题之一。学界已经提出了多种不同的自上而下方法来解决这一问题，例如弦理论、反德西特/共形场论 (AdS/CFT) 全息原理 (天体全息也可被理解为一种本质上微扰的自下而上的量子引力视角) 和圈量子引力 (下文简称 LQG)(该列表绝非完整列表，参见例如文献 [1])。尽管这些方法都取得了诸多成果，但它们基于根本不同的基础原理，且看起来无法调和。在我们看来，目前缺失的是一种自下而上的非微扰视角: 它能从我们对半经典引力及其对称性的理解出发，提供任何自上而下的量子引力方法都应当满足的坚实基础与检验。该视角包含全息原理的基础要素之一，即固定边界条件的引力系统的哈密顿量完全承载于其边界之上。它同时尊重 LQG 的核心基本原则之一——背景独立性，也符合等效原理的基础: 等效原理意味着引力动力学就是其对称性的体现。局域全息方法将全息原理与对称性视作同一枚硬币的两面。本篇综述我们重点介绍局域全息与 LQG 的关联，以及该框架得到的部分成果 (例如量子几何的出现)，暂不展开介绍它与传统全息的关联。

In particular, we show that gauge symmetry, which is usually understood as a mere redundancy of our physical description, is much more than that. It governs the quantum entanglement of subregions across entangling surfaces, called corners. These entangling surfaces are shown to carry representations of infinite-dimensional symmetry groups that are the physical consequence of the underlying gauge invariance. Moreover, we show that these symmetries represent, through their nontrivial commutation relations, the elements of quantum geometry.

我们特别要指出，通常被认为只是物理描述冗余的规范对称性，其意义远不止于此。规范对称性支配着纠缠面 (称为角区) across 子区域的量子纠缠。这些纠缠面承载了无限维对称群的表示，而这正是底层规范不变性的物理结果。此外我们还展示，这些对称性通过非对易关系表示了量子几何的基本单元。

This perspective is very close to some of LQG's most noticeable technical achievements, in particular, the rigorous construction of a Hilbert space of quantum Riemannian geometry based on a Hamiltonian formulation of general relativity in terms of SU(2) Yang-Mills gauge connection variables. In this context, symmetries such as spatial diffeomorphisms and local SU(2) frame rotations were central for defining the physical states. The emphasis on SU(2) gauge symmetry led to the quantum numbers labeling the kinematical spin network states [2].

这一视角与 LQG 最引人注目的部分技术成果非常契合，尤其是基于 SU(2) 杨-米尔斯规范联络变量对广义相对论做哈密顿表述后，严格构造出了量子黎曼几何的希尔伯特空间。在该体系中，空间微分同胚和局域 SU(2) 标架转动这类对称性对定义物理态至关重要。对 SU(2) 规范对称性的重视催生了标记运动学自旋网络态的量子数 [2]。

Since it was initially conceived as a canonical quantization of general relativity in the Hamiltonian formulation, LQG is often seen as a bulk quantization. Little to no role is played by boundary structures. One goal of the present review chapter is to explain why this is not so and how LQG accommodates the description of some boundary degrees of freedom in its foundations. For example, one of the key results of LQG is that spin networks represent a notion of quantum geometry. Suppose we cut open a portion of quantum geometry along a surface S . In that case, we obtain, for each intersection between the spin network and the surface S , an SU(2) representation, which carries an elementary quantum of geometry. Moreover, the SU(2) generators associated with each cut are the quantum flux operators integrated along S . These quanta, which are fundamentally entangled and represent quantum geometry, are controlled by gauge symmetry. One limitation of the traditional LQG approach is that it assumes a discretization of the fundamental geometrical excitations. Local holography allows us to keep some of the key successes of LQG, such as the quantization of the area spectra while being fully compatible with the standard understanding of QFT in the continuum. The presentation will be done in four main parts.

由于 LQG 最初被构想为哈密顿框架下广义相对论的正则量子化, 它通常被看作是体量子化方法, 边界结构几乎不发挥作用。本章综述的一个目标就是说明实际情况并非如此，以及 LQG 的基础其实可以容纳对部分边界自由度的描述。例如，LQG 的核心结论之一是自旋网络表示量子几何的概念。假设我们沿着曲面 S 切出一块量子几何区域，那么对于自旋网络与曲面 S 的每个交点，我们都会得到一个 SU(2) 表示，它承载了一份基本的几何量子。此外，与每个切口关联的 SU(2) 生成元就是沿 S 积分的量子通量算符。这些从根本上纠缠、代表量子几何的量子由规范对称性调控。传统 LQG 方法的一个局限是它假设基本几何激发是离散化的。局域全息可以保留 LQG 的部分核心成果，例如面积谱的量子化，同时完全兼容连续介质下量子场论的标准认知。本文将分为四个主要部分展开介绍。

In section ”Gauge Theories in Bounded Regions,” we first explain how the notion of boundary degrees of freedom arises in gauge theories and highlight the fundamental role that it plays in classical and quantum gravity. We develop these ideas further in section ”Quantum Geometry and Corner Symmetry” by explaining how boundary symmetries are related to entanglement and how their quantization gives rise to a notion of quantum geometry. This then motivates us to summarize the role played by boundary degrees of freedom in LQG. We recall in section ”Boundaries in Loop Quantum Gravity” various results establishing that spin network data can be seen as boundary data and also summarize earlier results on boundaries and entanglement in LQG. We then review recent results on the construction of corner charges in the first-order formulation of gravity in the presence of the Barbero-Immirzi parameter and give the proof of a Lorentz covariant area operator with discrete spectra. This section also reveals our shared viewpoint on future developments. It shows how the non-perturbative quantum gravity perspective started by LQG should be extended in order to fully provide a representation of the boundary symmetries compatible with subsystem decomposition and coarse-graining. Finally, in section ”Null Boundaries, Isolated Horizons, and Fluid Conservation Law”, we review our current understanding of the representation of super-translation symmetry, emphasizing the gravitational dynamics along null boundaries. We present our understanding of such dynamics as Carrollian charge conservation equations. We also construct a boundary action for finite null boundaries that encodes radiation and is compatible with discrete area spectra.

在“有界区域的规范理论”一节中，我们首先阐释规范理论中边界自由度的概念是如何产生的，并强调它在经典引力与量子引力中发挥的核心作用。我们在“量子几何与角对称性”一节进一步拓展这些思想，解释边界对称性如何与纠缠相关，以及对它们的量子化如何衍生出量子几何的概念。这进而推动我们总结边界自由度在圈量子引力 (LQG) 中发挥的作用。我们在“圈量子引力中的边界”一节回顾了多项结论，这些结论证实自旋网络数据可被视为边界数据，我们还总结了圈量子引力中关于边界与纠缠的早期研究成果。接着我们回顾了存在巴贝罗-伊米里兹参数时，一阶引力表述中构造角荷的最新研究结论，并给出了具有离散谱的洛伦兹协变面积算符的证明。本节还展现了我们对未来发展的共同观点。它说明了应当如何拓展由圈量子引力开创的非微扰量子引力视角，以得到完全兼容子系统分解与粗粒化的边界对称性表示。最后，在“类空边界、孤立视界与流体守恒律”一节，我们回顾了目前我们对超平移对称性表示的理解，重点讨论了沿零边界的引力动力学。我们将我们对这类动力学的理解表述为卡罗尔荷守恒方程。我们还为有限零边界构造了编码辐射且兼容离散面积谱的边界作用量。

Gauge Theories in Bounded Regions

有界区域中的规范场论

Generalized Noether Charges and Boundary Symmetries

广义诺特荷与边界对称性

In the case of gauge symmetries, Noether's first and second theorems together imply that charges are associated with codimension-2 boundaries. More precisely, to local gauge symmetries with parameter ε , one can assign the object

在规范对称性的情形下，诺特第一与第二定理共同表明，荷与余维数为 2 的边界关联。更准确地说，对参数为 ε 的定域规范对称性，我们可以赋予其如下对象

$$Q(\varepsilon)=\int_{\sum}C(\varepsilon)+\oint_{S}q(\varepsilon), \tag{1}$$

where $C(\varepsilon)\triangleq 0$ is the constraint and $q(\varepsilon)$ the so-called charge aspect. This result implies that the charges of local symmetries supported inside a region $\sum$ are given by an integral over its boundary $S=\partial\sum$. By definition, what then distinguishes a gauge transformation from a true physical symmetry is the possible non-vanishing value of the charge. We therefore see that in the presence of a boundary, whose intersection with $\sum$ is the corner S , gauge transformations can potentially become physical symmetries associated with a non-vanishing charge. Setting aside technical issue for the moment, one can think of the covariant phase space formalism as a method for assigning the charge (1) to an infinitesimal transformation δ_{ε} .

其中 $C(\varepsilon)\triangleq 0$ 是约束，$q(\varepsilon)$ 是所谓的荷分量。该结果表明，支撑在区域 $\sum$ 内部的定域对称性的荷，由区域边界 $S=\partial\sum$ 上的积分给出。根据定义，规范变换与真实物理对称性的区别就在于荷可能取非零值。因此我们可以看到，当存在边界且边界与 $\sum$ 相交于角区 S 时，规范变换有可能成为对应非零荷的物理对称性。暂不考虑技术问题，我们可以将协变相空间形式论理解为一种给无穷小变换 δ_{ε} 赋予荷 (1) 的方法。

Such surface charges are of fundamental importance as they describe essential physical observables. This is the case of the ADM mass, the angular momentum, the memory observables, the multipole moments, and, for example, the LQG fluxes. Moreover, the charges come equipped with certain algebraic structures: they form a local symmetry algebra, and their time development along the boundary can give rise to so-called flux/balance laws, describing, for example, the interplay between gravitational radiation and the failure of the charges to be conserved. It is indeed important to appreciate that the physically meaningful surface charges go beyond what one can be used to when studying Noether charges in mechanics. The charges can be non-conserved and also defined without actually resorting to boundary conditions, i.e., by relaxing the need for a variational principle. Moreover, the charges can also fail to be integrable in the variational sense.

这类面荷具有根本重要性，因为它们描述了核心物理可观测量。ADM 质量、角动量、记忆可观测量、多极矩，例如还有 LQG 流量都是这类面荷的例子。此外，荷还具备特定的代数结构: 它们构成一个定域对称代数，荷沿边界的时间演化可以产生所谓的流量/守恒律，描述比如引力辐射与荷不守恒之间的相互关联。认识到有物理意义的面荷超出了我们在力学中研究诺特荷的通常认知，这一点确实非常重要。荷可以不守恒，也可以无需借助边界条件来定义，即可以放松对变分原理的要求。此外，荷也可以不满足变分意义上的可积性。

For example, in the case of a diffeomorphism with parameter ξ , the charge takes the form

例如，对参数为 ξ 的微分同胚，荷的形式为

$$I_\xi \Omega = \delta Q(\xi) = \delta Q_{\mathrm{N}}(\xi) - \oint_S i_\xi \theta, \tag{2}$$

where Ω is the symplectic structure, obtained as the integral over $\sum$ of the variation $\delta\theta$ of the symplectic potential θ . This latter appears as the boundary term of the variation of the Lagrangian, i.e., $\delta L \triangleq \mathrm{d}\theta$. Finally, here $Q_{\mathrm{N}}(\xi)$ stands for the Noether charge associated with the Lie derivative $\mathcal{S}_\xi$. Importantly, this formula shows that the full surface charge is not only given by the Noether piece but can also contain a non-integrable flux contribution $F_N(\xi) = \oint_S i_\xi \theta$. More generally, the charge variations take the form

其中 Ω 是辛结构，由辛势 θ 的变分 $\delta\theta$ 在 $\sum$ 上积分得到。辛势本身作为拉格朗日量变分的边界项出现，即 $\delta L \triangleq \mathrm{d}\theta$ 。最后，此处 $Q_{\mathrm{N}}(\xi)$ 代表与李导数 $\mathcal{S}_\xi$ 关联的诺特荷。重要的是，这个公式表明完整的面荷不仅包含诺特分量，还可以包含不可积的流量贡献 $F_N(\xi) = \oint_S i_\xi \theta$ 。更一般地，荷的变分形式为

$$I_\varepsilon \Omega = \delta Q(\varepsilon) + F(\varepsilon), \tag{3}$$

where the right-hand side is a (non-unique) split between an integrable piece $Q(\varepsilon)$ and a flux contribution $F(\varepsilon)$. We leave aside technical discussions on how to select a preferred split of the charge between integrable and non-integrable parts, but a canonical choice is the Noetherian split, $Q(\varepsilon) = Q_{\mathrm{N}}(\varepsilon)$, which is entirely determined by the choice of boundary Lagrangian [3,4]. What is important is that a repeated symmetry action on a charge leads to the equation

其中右侧是可积分量 $Q(\varepsilon)$ 和流量贡献 $F(\varepsilon)$ 的 (不唯一) 分解。我们暂且不讨论如何在可积部分和不可积部分之间选择优先分解的技术问题，但标准选择是诺特分解 $Q(\varepsilon) = Q_{\mathrm{N}}(\varepsilon)$ ，它完全由边界拉格朗日的选择决定 [3,4]。重要的结论是，对称性对荷的重复作用给出如下方程

$$\delta_{\varepsilon_2} Q(\varepsilon_1) = Q([\varepsilon_1, \varepsilon_2]) + I_{\varepsilon_1} F(\varepsilon_2)$$

(4)

$$\text{evolution} = \text{conservation} + \text{dissipation}.$$

As indicated, this can be thought of as an evolution generated by the symmetry, which is composed of a rotation dictated by the symmetry algebra, and a possible dissipation due to the flux term. This latter is typical of open Hamiltonian systems, in which modes can escape, e.g., through the null boundary in the form of gravitational radiation. An important point of this construction explained in detail for the first time in [5] is that once we take into account the presence of edge mode fields, it is possible to show that the symmetry charges are gauge invariant. They commute with the Hamiltonian constraints. In Gravity, the edge mode fields are given by the choice of embedding maps from a reference ball to the spacetime [6, 7] . For simplicity, we do not introduce them in our presentation even if they play an important conceptual role.

> 如上所示，这可以理解为由对称性生成的演化，它包含对称代数决定的转动，以及流量项带来的可能耗散。耗散是开放哈密顿系统的典型特征，在这类系统中，模式可以逃逸，例如以引力辐射的形式穿过类光边界。该构造的一个重要要点 (首次在文献 [5] 中得到详细解释) 是: 一旦我们考虑边缘模场的存在，就可以证明对称荷是规范不变的，它们与哈密顿约束对易。在引力中，边缘模场由从参考球到时空 [6, 7] 的嵌入映射给出。为简化表述，即使它发挥着重要的概念性作用，我们在本次阐述中也不引入它。

This structure suggests that there is both kinematical and dynamical information in the charges and their algebra. Kinematical charges are integrable and associated with a vanishing flux, while the dynamics amounts to restoring the flux and taking into account the radiative modes. The kinematical charges belong to the so-called corner symmetry group, which is a subgroup $G_0^S \subset G_{\text{ext}}^S$ of the full symmetry group. The corner symmetry charges form a closed algebra $\{Q_0(\varepsilon_1), Q_0(\varepsilon_2)\} = Q_0([\varepsilon_1, \varepsilon_2])$ which can be readily quantized. The full symmetry group is called the extended corner symmetry group. The extended corner symmetry charges can only be understood as canonical symmetry charges after extending the gravitational phase space with an edge mode field called the embedding field [6, 7] . More recent work [8] connects this research to the geometry of open systems, dissipation, and entropy production via the framework of metriplectic geometry. At the quantum level, physical observables should provide representations of G_0^S and G_{ext}^S . This is a necessary condition in order to have a quantization compatible with the symmetries of the underlying classical theory. In general relativity, as we will show below, representing the corner charge algebra amounts to a quantization of geometry, while representing the full dynamical charge algebra is expected to provide a notion of quantum dynamics. For a demonstration of this in the context of asymptotic infinity, see [9].

> 这一结构表明，诺特荷及其代数中同时包含运动学与动力学信息。运动学荷是可积的，对应零通量，而动力学则对应恢复通量并考虑辐射模式。运动学荷属于所谓的角对称性群，它是整个对称性群的一个子群 $G_0^S \subset G_{\text{ext}}^S$ 。角对称荷构成一个可直接量子化的闭代数 $\{Q_0(\varepsilon_1), Q_0(\varepsilon_2)\} = Q_0([\varepsilon_1, \varepsilon_2])$ 。整个对称性群被称为扩展角对称性群。只有当我们通过一个称为嵌入场的边模场 [6, 7] 扩展引力相空间后，才能将扩展角对称荷理解为正则对称荷。近期的文献 [8] 通过度量几何框架将该研究与开放系统几何、耗散和熵产生联系起来。在量子层面，物理可观测量应为 G_0^S 和 G_{ext}^S 提供表示。这是实现与基础经典理论对称性兼容的量子化的必要条件。正如我们下文将要展示的，在广义相对论中，表示角电荷代数等价于几何的量子化，而表示完整动力学电荷代数则有望给出量子动力学的定义。有关渐近平直背景下的相关论证，参见文献 [9]。

What we have left out of the discussion so far is an explanation of the zoology of charges and charge algebras which can arise. This depends, of course, on (i) the theory being considered (e.g., gravity or Maxwell), (ii) the symmetry transformations being studied (e.g., diffeomorphisms or internal Lorentz transformations), (iii) the type of boundary conditions (e.g., conservative or radiative boundary conditions), (iv) the nature of the boundary (e.g., an asymptotic boundary or an entangling surface at a finite distance), and (v) the choice of boundary Lagrangian or topological terms such as theta terms. Recent years have seen a wealth of developments and new results on the covariant phase space which has fueled lots of the developments we present in this survey. In addition to the references [3- 7] cited above, the interested reader should consult [10-15]. Moreover, the recent report view [16] contains a comprehensive review of the ideas we presented here and many more references.

我们目前的讨论尚未涵盖可能出现的各类电荷与电荷代数。这当然取决于:(i) 所研究的理论 (例如引力或麦克斯韦理论)，(ii) 所研究的对称变换 (例如微分同胚或内部洛伦兹变换)，(iii) 边界条件的类型 (例如保守或辐射边界条件)，(iv) 边界的性质 (例如渐近边界或有限距离处的纠缠面)，以及 (v) 边界拉格朗日量或 θ 项这类拓扑项的选择。近年来协变相空间领域取得了大量进展与新成果，推动了本文综述中介绍的诸多研究。除上述引用的文献 [3-7] 外，感兴趣的读者可查阅文献 [10-15]。此外，近期综述文献 [16] 包含了对本文所述思想的全面综述，并提供了更多参考文献。

In order to make contact with LQG, we will focus below on gravity in tetrad variables and consider the corner symmetries and charges which can be derived for any quasi-local region (i.e., a boundary at a finite distance) and without the need for boundary conditions. An immediate example of such a charge is that obtained from (2) in the case of tangential vector fields, for which the flux term vanishes (One should distinguish between symplectic flux, which can exist at a cut, from the flux due to time evolution. In the case of tangential vector fields, the former is absent, but of course there is still a non-trivial flux of, e.g., angular momentum due to its time evolution along $\mathcal{I}$. There is therefore a translation flux but no rotation flux. In equation (4), one can indeed see that even if ε_1 is tangent, then there is still a non-trivial flux due to time evolution if ε_2 is an infinitesimal translation in the u direction.).

为了与圈量子引力 (LQG) 建立联系，我们下文将重点讨论余标架形式下的引力，研究可从任意准局域区域 (即有限距离处的边界) 推导得到、无需额外边界条件的角对称性与荷。这类荷的一个直接例子由切矢量场情形下的式 (2) 得到，该情形下通量项为零 (我们需要区分切口处可能存在的辛通量与时间演化产生的通量。对于切矢量场，不存在前者，但沿 $\mathcal{I}$ 做时间演化时，角动量这类量仍会存在非平庸通量，因此存在平移通量但不存在旋转通量。在式 (4) 中可以清楚看到，即使 ε_1 是切矢量，若 ε_2 是 u 方向上的无穷小平移，时间演化仍会产生非平庸通量。)。

Implications for Classical and Quantum Gravity

经典引力与量子引力的启示

The role of symmetries in the study of a classical and quantum theory cannot be understated. Provided enough conserved quantities exist in a theory, they can be used to solve the dynamics altogether. Although we do not expect this to be the case in gravity, it is still the case that symmetries provide a powerful organizational tool and give the possibility of controlling the quantization of geometry in terms of symmetry observables.

对称性在经典理论和量子理论研究中的作用怎么强调都不为过。如果一个理论中存在足够多的守恒量，就可以用它们直接求解整个动力学。虽然我们不期待引力也满足这一点，但对称性依然是强有力的结构化工具，并且有望通过对称性可观测量来约束几何的量子化。

In the case of asymptotic symmetries for asymptotically flat spacetimes, the relationship between corner symmetries (described by the so-called BMS group and its generalizations [17, 18]), classical observables, and the structure of the perturbative quantum theory has already been demonstrated and is being used to build a notion of flat space holography (see [19-21] and the references therein). This idea of building the quantum theory on the structure of the classical symmetries is also what gave rise to the holographic AdS/CFT correspondence. It is clear that the very existence of boundary charges and symmetries in the classical theory should play a central role in any attempt to quantize gravity and understand what form of holography holds in quantum gravity.

在渐近平直时空的渐近对称性情形中，顶角对称性 (由所谓 BMS 群及其推广描述 [17, 18])、经典可观测量与微扰量子理论结构之间的关联已经得到证明，目前正被用于构建平空间全息论的概念 (见 [19-21] 及其中的参考文献)。这种基于经典对称性结构构建量子理论的思路，也正是全息 AdS/CFT 对偶的起源。很明显，在经典理论中边界荷与对称性的存在，对于任何量子引力的量子化尝试、理解量子引力中成立的全息论形式而言，都应当发挥核心作用。

Quantum Geometry and Corner Symmetry

量子几何与角对称性

The viewpoint just presented helps understand the fundamental tension which exists between gravity and local quantum field theory. In any gauge theory, the presence of constraints implies that gauge-invariant observables are necessarily non-local. This means, in turn, that the Hilbert space does not factorize spatially, as factorization would leave out of the sub-factors some of the non-local observables which cut across the entangling surface (see [22-28] for many discussions on this issue in gauge theory).

刚刚介绍的观点有助于理解引力与局域量子场论之间存在的根本性张力。在任何规范理论中，约束的存在意味着规范不变可观测量必然是非局域的。这反过来意味着希尔伯特空间无法按空间分解，因为分解会将部分穿过纠缠面的非局域可观测量排除在子因子之外 (关于规范理论中该问题的诸多讨论参见 [22-28])。

Entanglement from Symmetry

对称性产生的纠缠

Let us now explain more in detail why corner symmetries are essential to unravel the nature of spacetime entanglement and lead to a covariant description of quantum spacetime that generalizes the one given by LQG. The key idea proposed in [5] is that corner symmetry allows us to resolve the quantum entanglement of spacetime through symmetries derived from the gauge principle. This correspondence between symmetry and entanglement was first realized in the context of lattice gauge theory in [25].

现在我们来更详细地解释，为什么顶角对称性对于揭示时空纠缠的本质至关重要，并且能给出推广了圈量子引力 (LQG) 的量子时空协变描述。文献 [5] 提出的核心观点是: 顶角对称性允许我们通过从规范原理导出的对称性来分解时空的量子纠缠。这种对称性与纠缠之间的对应关系最早是在文献 [25] 的格点规范论框架中被发现的。

Let us start with an example in the standard field theory, and let us focus on the vacuum state $|0\rangle$. This state is an eigen state not only of the Hamiltonian H but also of the boost operator K that fixes the two-dimensional plane $S=\left\{x^3=0\right\}$ inside the $x^0=$ const. Cauchy slice. In other words we have

我们先从标准场论中的一个例子开始，聚焦于真空态 $|0\rangle$ 。该态不仅是哈密顿量 H 的本征态，也是固定了 $x^0=$ 常数柯西切片内二维平面 $S=\left\{x^3=0\right\}$ 的快度 boost 算子 K 的本征态。换言之我们有

$$H\,|0\rangle=K|\,0\rangle=0. \tag{5}$$

The plane S fixed by K is called the entangling surface. It divides the slice into a left and a right component $\sum=\sum_L\cup\sum_R$ with $S=\partial\sum_{L/R}$. One can decompose the Hilbert space as a product of left (respectively, right) Hilbert spaces (Strictly speaking this decomposition is only valid at the level of the observable algebras due to UV divergences. See [29] for the proper description. For simplicity, we keep the naive language of Hilbert space factorization, valid for a UV-regulated QFT.) obtained by acting on the vacuum by local operators supported on $\sum_L$ (respectively, $\sum_R$). Tracing out the left Hilbert space, one obtains the density matrix $\rho_S=\mathrm{Tr}_{H_L}\left(|0\rangle\langle 0|\right)$.

被 K 固定的平面 S 称为纠缠面。它将切片分为左右两个部分 $\sum=\sum_L\cup\sum_R$ ，满足 $S=\partial\sum_{L/R}$ 。我们可以将希尔伯特空间分解为左右希尔伯特空间的乘积 (严格来说，由于紫外发散，这个分解仅在可观测量代数层面成立，正确的描述见文献 [29]。为简化表述，我们沿用希尔伯特空间分解的朴素表述，这在经过紫外正则化的量子场论中是成立的)，通过 $\sum_L$ (左侧)、$\sum_R$ (右侧) 上支撑的局域算子作用在真空上得到。对左希尔伯特空间求迹后，我们得到密度矩阵 $\rho_S=\mathrm{Tr}_{H_L}\left(|0\rangle\langle 0|\right)$.

Since the entangling cut is fixed by K and the slice is defined at $t=0$, we can decompose the total boost operator into a local sum $K=K_R-K_L$, where $K_{R/L}=\int_{\sum_{R/L}}|x^3|\,T_{00}$ is the right/left boost operator, also called the "twist operator," which commutes with local operators placed on the left side. $K_{R/L}$ is a positive operator, and it acts non-trivially on the vacua. This means that we can decompose the density matrix in terms of eigenstates $|n\rangle$ of the "twist operator" K_R with eigenvalue $K_n>0$.

由于纠缠切面由 K 固定,且切片定义在 $t=0$ 处,我们可以将总快度算子分解为局域求和 $K=K_R-K_L$ ，其中 $K_{R/L}=\int_{\sum_{R/L}}|x^3|\,T_{00}$ 是左右侧的快度算子，也称为“扭算子”，它与支撑在左侧的局域算子对易。$K_{R/L}$ 是正算子，对真空有非平凡作用。这说明我们可以将密度矩阵按扭算子 K_R 对应本征值 $K_n>0$ 的本征态 $|n\rangle$ 展开。

We can now use these symmetry generators to construct the symmetry-reduced state which are defined as the projection of ρ_S onto the eigenspaces of K_R . The main theorem of Unruh-Bisognano-Wichmann [30] states that the symmetry-reduced vacuum is pure. The second statement of this theorem is that this state is simply given by a complex boost associated with an imaginary boost angle of $2i\pi$. Let us recall that $\mathrm{U}(\tau)=e^{\frac{i}{\hbar}\tau K_R}$ denotes the action of the boost operator with boost angle τ on states supported on $\sum_R$. The Unruh-Bisognano-Wichmann result therefore simply states that

现在我们可以利用这些对称性生成元，构造对称性约化态——即 ρ_S 到 K_R 本征空间的投影。昂鲁-比索尼亚诺-威克曼主定理 [30] 指出，对称性约化真空是纯态。该定理的第二个结论是，这个态简单对应一个带有虚快度角 $2i\pi$ 的复快度变换。回顾一下，$\mathrm{U}(\tau)=e^{\frac{i}{\hbar}\tau K_R}$ 表示快度角为 τ 的快度算子作用在支撑于 $\sum_R$ 的态上。因此昂鲁-比索尼亚诺-威克曼的结论可简单表述为

$$\rho_S = e^{-\frac{2\pi}{\hbar}K_R}. \tag{6}$$

This means that the vacuum state can be decomposed as

这说明真空态可以分解为

$$\left|0\right\rangle = \sum_n e^{-\frac{\pi}{\hbar}K_n}\left|n, \tilde{n}\right\rangle, \tag{7}$$

where $|n, \widetilde{m}\rangle$ denotes an eigen state of K_R (respectively, K_L) with eigenvalue K_n (respectively, K_m). The lesson we learn from this example is that the vacuum entanglement is entirely contained from the matching of boost charges $K_R = K_L$ across the entangling surface. This is the expression of boost invariance, which implies that the state can be written as $\sum\limits_n \rho_n|n, \tilde{n}\rangle$ in terms of a measure ρ_n . We also learn that the form for this measure is determined by the demand of translation invariance of the state.

其中 $|n, \widetilde{m}\rangle$ 表示 K_R (对应 K_L) 本征值为 K_n (对应 K_m) 的本征态。我们从该例得到的结论是: 真空纠缠完全由纠缠界面上快度 boost 荷 $K_R = K_L$ 的匹配决定。这是快度不变性的体现，它说明该态可以用测度 ρ_n 表示为 $\sum\limits_n \rho_n|n, \tilde{n}\rangle$ 。我们还可以得知，该测度的形式由态的平移不变性要求确定。

So far we have reviewed a well-known result about quantum field theory (QFT) in the presence of an entangling surface. How does this relate to quantum gravity and corner symmetries? In QFT we saw that the only state to which we can apply the previous symmetry argument to compute entanglement is the vacuum state for which $K|0\rangle = 0 = H|0\rangle$. In gravity we know, as reviewed in the previous section, that on a closed Cauchy surface the Hamiltonian generator $Q(\xi)$ vanishes for any vector field ξ (A weaker statement would be that $\sum$ has an asymptotic boundary but ξ vanishes sufficiently fast near infinity.). At the quantum level, this means that $Q(\xi)|\Psi\rangle = 0$ for all quantum states of quantum gravity. Let us now choose a two-dimensional finite closed surface $S \in \sum$ which plays the role of an entangling surface and denote $\sum_L$ the surface inside S and $\sum_R$ the slice outside S . Let us then consider the set of vector fields ξ which do not move S . We can then decompose the diffeomorphism charge in terms of its left and right components $Q(\xi) = Q_R(\xi) - Q_L(\xi)$. As we have seen in the previous section, $Q(\xi)$ is the integral of the Hamiltonian constraint on $\sum$, while the charges $Q_R(\xi)$ are given by corner integrals and the corner symmetry algebra $\mathfrak{g}^S$ is the sub-algebra of infinitesimal diffeomorphisms fixing S that possess a non-vanishing charge corner charge $Q_R(\xi) = \oint_S q_\xi$. We are thus in a situation that is analogous to the Unruh case. We expect that the physical Hilbert space splits into a sum of representations ρ of the corner symmetry group. In other words, given the existence of a corner symmetry algebra, the quantum gravity states Ψ can be decomposed into a sum

到目前为止，我们回顾了量子场论 (QFT) 中存在纠缠界面时的一个经典结论。这与量子引力和角对称性有何关联？在量子场论中我们已经看到，只有真空态能够应用前述对称性论证计算纠缠，该态满足 $K|0\rangle = 0 = H|0\rangle$。在引力中，如我们在前一节回顾的，在闭合柯西面上，对任意矢量场 ξ，哈密顿生成元 $Q(\xi)$ 都为零 (弱一点的表述是: $\sum$ 存在渐近边界，但 ξ 在无穷附近衰减足够快)。在量子层面，这意味着对于量子引力的所有量子态都满足 $Q(\xi)|\Psi\rangle = 0$。现在我们选取一个二维闭合有限曲面 $S \in \sum$ 作为纠缠界面，记 $\sum_L$ 为 S 内部的曲面，$\sum_R$ 为 S 外部的切片。接下来我们考虑一类不移动 S 的矢量场 ξ，我们可以将微分同胚荷分解为左右分量 $Q(\xi) = Q_R(\xi) - Q_L(\xi)$。正如我们在前一节所见，$Q(\xi)$ 是 $\sum$ 上哈密顿约束的积分，而荷 $Q_R(\xi)$ 由角积分给出，角对称性代数 $\mathfrak{g}^S$ 是固定 S 的无穷小微分同胚的子代数，这类微分同胚拥有非零角荷 $Q_R(\xi) = \oint_S q_\xi$。因此我们现在得到的情形与安鲁效应的情形类似。我们预期物理希尔伯特空间可以分解为角对称群的表示 ρ 之和。换句话说，由于角对称性代数存在，量子引力态 Ψ 可以分解为

$$\Psi = \sum_\rho \psi_\rho(S) \sum_a |\rho, a\rangle \otimes |\rho^*, a\rangle.| \tag{8}$$

Here $\rho*$ denotes the dual representation and a denotes a label of states in the Hilbert space V_ρ. This decomposition can be extended to more than one surface $S \to (S_1, \cdots S_n)$ and more generally to a collection of cuts Γ that represents a two-dimensional cellular complex. The previous calculation of charges as corner charges generalizes to this case. On the two-dimensional cells of this complex, we assign representations of the corner symmetry group, and on the one-dimensional cells which glue the two-cells, we assign intertwiners of the corner symmetry algebra. At the end of the day, we can therefore expect a decomposition of the quantum gravity state as a sum

此处 $\rho*$ 表示对偶表示，a 表示希尔伯特空间 V_ρ 中的态标签。该分解可以推广到多个曲面 $S \to (S_1, \cdots S_n)$，更一般地可以推广到表示二维胞腔复形的一组割 Γ。前述将荷作为角荷的计算可以推广到该情形。在复形的二维胞上，我们赋予角对称群的表示；在粘合二维胞的一维胞上，我们赋予角对称性代数的缠结算子。因此我们最终可以预期，量子引力态可以分解为

$$|\Psi\rangle = \sum_{\rho_\Gamma} \psi_{\rho_\Gamma}(\Gamma) \sum_{a_\Gamma} |\rho_\Gamma, a_\Gamma\rangle \otimes |\rho_\Gamma^*, a_\Gamma\rangle. \tag{9}$$

Here $\rho_\Gamma = \left(\otimes_f \rho_f, \otimes_e I_e\right)$, where f labels the faces of Γ with representations ρ_f, and e labels the edges of the complex with intertwiners I_e. This decomposition must be valid for any quantum gravity state.

在此处 $\rho_\Gamma = \left(\otimes_f \rho_f, \otimes_e I_e\right)$，其中 f 标记了 Γ 带有表示 ρ_f 的面，e 标记了该复形中带有缠结算子 I_e 的边。该分解对任意量子引力态都必须成立。

This decomposition is reminiscent of the spin network decomposition of quantum gravity states that appears in loop quantum gravity (See [31] or the chapter [2] in this volume.), but it is also more general. The difference is that it appears in a continuum description and the group that one needs to represent is the full corner symmetry group (and not only the SU(2) group of internal frame rotations as in

这种分解让人联想到圈量子引力中出现的量子引力态自旋网络分解 (见 [31] 或本卷中的第二章 [2])，但它的适用范围更广泛。区别在于，该分解是在连续描述中给出的，且需要表示的群是完整的边界对称群 (而不只是像

LQG). As we are about to see, in the corner symmetry proposal, the quantization of geometry appears through the choice of representation and through quantization of the spectra of the geometric operators that represents the corner symmetry charges. The central questions of quantum gravity now become, in this context: what are the representations that enter the decomposition of the quantum gravity states and what is the integration measure Ψ_ρ that plays the role of the Unruh measure $e^{\frac{\pi K_n}{\hbar}}$ for quantum gravitational states?

LQG 中那样仅为内部参考架转动的 SU(2) 群)。我们接下来就会看到，在边界对称方案中，几何的量子化是通过选择表示、以及通过表示边界对称荷的几何算符的谱量子化实现的。在此背景下，量子引力的核心问题现在变为: 哪些表示会参与量子引力态的分解，以及对量子引力态起到安鲁测度 $e^{\frac{\pi K_n}{\hbar}}$ 作用的积分测度 Ψ_ρ 是什么?

Representation Theory of Corner Symmetries

角对称性表示论

In metric gravity it has been shown that the corner symmetry group (The extended corner symmetry group is obtained by adding the super-translations along the two normal directions and is given by [5, 32]

在度量引力中，已证明角对称群 (扩展角对称群可通过添加沿两个法向方向的超平移得到，形式为 [5, 32]

$$G^S_{\text{metric, ext}} = \left(\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S\right) \ltimes \mathbb{R}^{2S}.) \tag{10}$$

is simply the semi-direct product [5] $G^S_{\text{metric}} = \mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S$, where $\mathrm{SL}(2,\mathbb{R})^S$ denotes the space of functions from the corner S to $\mathrm{SL}(2,\mathbb{R})$. These represent the boost transformations preserving S , while $\mathrm{Diff}(S)$ are the sphere diffeomorphisms. In the first-order tetrad (or frame) formalism, the symmetry group at the corner is extended by $\mathrm{SL}(2,\mathbb{C})^S$, namely the group of internal local Lorentz transformations supported on the sphere [33]. In the tetrad formulation, the corner symmetry group is therefore

正是半直积 [5] $G^S_{\text{metric}} = \mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S$ ，其中 $\mathrm{SL}(2,\mathbb{R})^S$ 表示从角 S 到 $\mathrm{SL}(2,\mathbb{R})$ 的函数空间。这些函数表示保持 S 的快度变换，而 $\mathrm{Diff}(S)$ 是球面微分同胚。在一阶标架 (即框架) 形式中，角处的对称群被 $\mathrm{SL}(2,\mathbb{C})^S$ 扩展，即支撑在球面上的局域洛伦兹变换群 [33]。因此在标架表述中，角对称群为

$$G^S_{\text{frame}} = \mathrm{Diff}(S) \ltimes \left(\mathrm{SL}(2,\mathbb{R})^S \times \mathrm{SL}(2,\mathbb{C})^S\right). \tag{11}$$

The study of the representations of these groups has been initiated in the work [34] and developed further in [33,35-37]. To connect this general discussion to the traditional discussion in LQG, one can go back to the work [38], which proposed the study of the loop corner symmetry group

这些群表示的研究由文献 [34] 发起，并在 [33,35-37] 中得到进一步发展。为了将该一般性讨论与圈量子引力 (LQG) 中的传统讨论联系起来，可追溯到文献 [38]，该文献提出研究圈角对称群

$$G^S_{\mathrm{LQG}} = \mathrm{Diff}(S) \ltimes \mathrm{SU}(2)^S, \tag{12}$$

obtained in the study of the canonical formulation of gravity in the time gauge. This is the subgroup of G^S_{frame} preserving the slice normal at S. In this context, the gravitational phase space is described in the bulk in terms of the SU(2) valued Ashtekar-Barbero connection A^i and in terms of the 2-form flux element $E_i = \frac{1}{2}\varepsilon_{ijk}\left(e^j \wedge e^k\right)$, where $e^i = \mathrm{d}x^a e^i_a$ denotes the frame field of the induced metric on $\sum$. Before imposing the constraints, these variables are canonically conjugated as

它得自时间规范下引力正则形式的研究，是保持 S 处切片法向的 G^S_{frame} 子群。在此背景下，引力相空间在体中由取值为 SU(2) 的阿西特卡-巴贝罗联络 A^i 和 2 形式流元 $E_i = \frac{1}{2}\varepsilon_{ijk}\left(e^j \wedge e^k\right)$ 描述，其中 $e^i = \mathrm{d}x^a e^i_a$ 表示 $\sum$ 上诱导度量的标架场。施加约束之前，这些变量满足正则对易关系:

$$\left\{A^i_a(x), E^b_j(y)\right\} = \delta^b_a \delta^i_j \delta^{(3)}(x-y). \tag{13}$$

The kinematical constraint equations associated with diffeomorphism and SU(2) gauge symmetry are simply, as shown in [38], the Gauss conservation laws

如文献 [38] 所示，微分同胚和 SU(2) 规范对称性对应的运动学约束方程就是高斯守恒律

$$\mathrm{d}_A E_i \triangleq 0, \quad \mathrm{d}_A P_i \triangleq 0. \tag{14}$$

Here E_i is the geometrical 2-form flux, while $P^i := \mathrm{d}_A e^i$ is the torsion of the Ashtekar-Barbero connection. On the reduced phase space, P^i computes components of the extrinsic curvature tensor. One can also understand P_i as the variable conjugated to the frame field since

此处 E_i 是几何 2 形式流，$P^i := \mathrm{d}_A e^i$ 是阿西特卡-巴贝罗联络的挠率。在约化相空间上，P^i 可计算外曲率张量的分量。也可将 P_i 理解为与标架场共轭的变量，因为

$$\left\{P^a_i(x), e^j_b(y)\right\} = \delta^a_b \delta^i_j \delta^{(3)}(x-y) \tag{15}$$

on the kinematical phase space.

该关系在运动学相空间上成立。

It is important to appreciate that the commutation relations (13), or the one just shown above, are only valid before the imposition of the constraints. What matters at the quantum level is to understand how these variables commute in the reduced phase space obtained after imposing the constraints. This is where the corner symmetry enters as a key ingredient. It is direct to show [38, 39] that the symmetry charge of infinitesimal diffeomorphisms labelled by a vector $\xi^a \partial_a \in T\sum$ which is tangent to S and of infinitesimal SU (2) gauge transformation labelled by $\alpha^i \sigma_i \in \mathfrak{su}(2)$ is, respectively, given by the corner charges

需要明确:(13) 式的对易关系，即上文给出的对易关系，仅在施加约束前有效。量子层面的核心问题是理解这些变量在施加约束后得到的约化相空间中的对易关系，此时角对称性就是关键要素。可直接证明 [38, 39] : 由切于 S 的矢量 $\xi^a\partial_a \in T\sum$ 标记的无穷小微分同胚， 以及由 $\alpha^i\sigma_i \in \mathfrak{su}(2)$ 标记的无穷小 SU (2) 规范变换，它们的对称荷分别由角荷给出

$$P(\xi) \triangleq \oint_S \xi^a e_a^i P_i,\ G(\alpha) \triangleq \oint_S E_i \alpha^i, \tag{16}$$

where the hatted equality emphasizes that these equations are valid on the reduced phase space.

其中带帽的等号强调这些方程在约化相空间上成立。

In particular one sees that, after the imposition of the Gauss constraints, the pullback on S of the electric field operator satisfies ultra-local commutation relations at the corner is given by [5, 40]

特别地可以看到，施加高斯约束后，电场算子拉回至 S 在角处满足超局域对易关系，形式为 [5, 40]

$$\left\{E^i(\sigma), E^j(\sigma')\right\}_S \triangleq \varepsilon^{ij}{}_k \delta^{(2)}(\sigma - \sigma') E^k(\sigma), \tag{17}$$

where $\sigma, \sigma' \in S$. Usually such a non-trivial commutation relation of the flux operator is only obtained in LQG after the introduction of loops which discretize the support of the flux lines. What is remarkable is that we can establish this non-commutation directly in the continuum and at the classical level through the Noether theorem. Similarly, the momentum operators $P_A = P_i e_A^i$ with $A = 1, 2$ indices tangent to S are the generators of diffeomorphisms along S . The commutation relations of these momenta on the reduced phase space are

其中 $\sigma, \sigma' \in S$ 。通常，只有在 LQG 中引入环路对流量线的支集离散化后，才能得到流量算符这种非对易关系。值得注意的是，我们可以在经典层面的连续体中直接通过诺特定理建立这种非对易关系。同理，带有切于 S 的 $A = 1, 2$ 指标的动量算符 $P_A = P_i e_A^i$ 是沿 S 的微分同胚生成元。这些动量在约化相空间上的对易关系为

$$\{P_A(\sigma), P_B(\sigma')\} \triangleq P_A(\sigma')\partial_B \delta^{(2)}(\sigma, \sigma') - P_B(\sigma)\partial'_A \delta^{(2)}(\sigma, \sigma'). \tag{18}$$

At the quantum level, the goal is to represent these commutation relations. The commutation relations (17) for the flux fields define a 2-sphere generalization of an SU (2) valued loop algebra. The representations are classified by the $\mathrm{SU}(2)^S$ whose Casimir is given by the corner area element $\rho(\sigma) = \left|\det\left(e_a^i\right)\right|(\sigma)$. At the classical level, this object defines a measure on the sphere which is strictly positive.

在量子层面，我们的目标是表示这些对易关系。流量场的对易关系 (17) 定义了一种 SU (2) 值环路代数的二维球面推广。表示由 $\mathrm{SU}(2)^S$ 分类，其卡西米尔量由角面积元 $\rho(\sigma) = \left|\det\left(e_a^i\right)\right|(\sigma)$ 给出。在经典层面，该对象定义了球面上一个严格正的测度。

At the quantum level, we have to choose a representation of the operators we just described, that is, we have to choose a unitary representation of G_{LOG}^S . These representations are classified by the choice of sphere measure. We now focus our analysis to representations labelled by a discrete measure on S which now carry a new quantum number N : the number of punctures on the sphere where this measure is non-vanishing. The representation states of $\mathrm{SU}(2)^S$ are given by functions $\varphi : S^N \to \otimes_{i=1}^N V_{j_i}$, where V_j denotes a spin- j representation of SU (2) and N denotes the number of punctures activated on the corner sphere. On these states, the action of the flux operator is then simply given by

在量子层面，我们需要选择刚才描述的算符的表示，即需要选择 G^S_{LOG} 的一个幺正表示。这些表示由球面测度的选择分类。我们现在将分析聚焦于由 S 上离散测度标记的表示，这类表示带有一个新量子数 N : 即该测度非零的球面上的穿孔数。$\mathrm{SU}(2)^S$ 的表示态由函数 φ 给出: $S^N \to \otimes_{i=1}^N V_{j_i}$ ，其中 V_j 表示 SU (2) 的自旋- j 表示，N 表示角球面上激活的穿孔数。流量算符在这些态上的作用可简单表示为

$$E_i(\sigma)\,\varphi(\sigma_1,\cdots\sigma_N) = \sum_{n=1}^{N} \delta^{(2)}(\sigma,\sigma_n)\,\rho_{j_n}(X_i)\,\varphi(\sigma_1,\cdots\sigma_N)\,, \tag{19}$$

where X_i is an SU (2) generator and $\rho_j : \mathfrak{su}(2) \to V_j$ denote the spin- j representations. This representation corresponds to a choice of discrete density given by

其中 X_i 是一个 SU (2) 生成元，$\rho_j : \mathfrak{su}(2) \to V_j$ 表示自旋- j 表示。该表示对应于如下离散密度选择:

$$\hat{\rho}(\sigma)\,\varphi(\sigma_1,\cdots,\sigma_N) = \sum_{n=1}^{N} \rho_n \delta^{(2)}(\sigma,\sigma_n)\,\varphi(\sigma_1,\cdots,\sigma_N)\,, \tag{20}$$

where $\rho_n = \gamma\hbar\sqrt{j_n(j_n+1)}$ is the discrete area spectrum of LQG.

其中 $\rho_n = \gamma\hbar\sqrt{j_n(j_n+1)}$ 是 LQG 的离散面积谱。

An interesting analogy comes by noticing that $G^S_{\mathrm{hvdro}} = \mathrm{Diff}(S) \ltimes \mathbb{R}^S$, which is the group generated by (P_A, ρ) , is isomorphic to the symmetry group of a two-dimensional barotropic fluid (Barotropic fluids are such that the fluid pressure $P(\rho)$ is a function of the fluid density ρ only. In general the pressure can also depend on the entropy density.) [41]. In this analogy, ρ is the fluid particle density, and it is well known that there exist two classes of fluid representations: the ones for which ρ is a strictly positive continuous measure, and the ones for which ρ is a discrete measure. In the second case the fluid is composed of molecules. Here we see that the quantization of area is analog to having a corner quantum fluid made of fundamental quanta.

一个有趣的类比是，我们注意到由 (P_A, ρ) 生成的群 $G^S_{\mathrm{hvdro}} = \mathrm{Diff}(S) \ltimes \mathbb{R}^S$ 同构于二维正压流体的对称群 (正压流体指流体压强 $P(\rho)$ 仅依赖于流体密度 ρ 。一般情况下压强还可以依赖于熵密度。)[41]。在这个类比中，ρ 是流体粒子密度，且众所周知流体表示存在两类: 一类中 ρ 是严格正的连续测度，另一类中 ρ 是离散测度。第二类流体由分子组成。在此我们看到，面积量子化类似于角量子流体由基本量子构成。

This analogy also allows us to classify the representations of Diff(S). It is known in the mathematical physics literature that these groups are non-anomalous [42]. This is why we can assume that the quantum theory provides a representation for these groups.

这个类比也能帮助我们对 Diff(S) 的表示分类。数学物理文献中已知这些群是非反常的 [42]。这就是我们可以假设量子理论为这些群提供了一个表示的原因。

As shown in [9], the representation of the diffeomorphism symmetry group is then labelled by conformal dimensions $(\Delta_1, \cdots, \Delta_N)$ and spins $(s_1, \cdots, s_N)$. These representations are irreducible and also appear as representation labels for BMSW. The Casimirs associated with diffeomorphism have been constructed in

正如文献 [9] 所示，微分同胚对称群的表示由共形维度 $(\Delta_1, \cdots, \Delta_N)$ 和自旋 $(s_1, \cdots, s_N)$ 标记。这些表示是不可约的，也可作为 BMSW 的表示标记。微分同胚对应的卡西米尔已构造于

[34]. The action of the diffeomorphism generator P_A on the discrete representation states is then given by

文献 [34]。微分同胚生成元 P_A 对离散表示态的作用可表示为

$$\widehat{P}_A(\sigma)\varphi(\sigma_1, \cdots, \sigma_N) = \sum_{n=1}^{N} \delta^{(2)}(\sigma, \sigma_i) \frac{\partial \varphi}{\partial \sigma_n^A}(\sigma_1, \cdots, \sigma_N)$$

$$-\sum_{n=1}^{N} \left(\Delta_n \delta_A{}^B + s_n \varepsilon_A{}^B\right) \frac{\partial \delta^{(2)}(\sigma, \sigma_n)}{\partial \sigma^B} \varphi(\sigma_1, \cdots, \sigma_N).$$

(21)

Here $\varepsilon_A{}^B$ corresponds to the choice of a complex structure on the sphere, i.e., it satisfies $\varepsilon_A{}^B \varepsilon_B{}^C = -\delta_A{}^C$. We see that a generic discrete representation of the LQG corner symmetry group assigns three quantum numbers (j, Δ, s) to each puncture. The states associated with (j, Δ, s) can be expanded as a superposition of spin network states purely labeled by SU(2) indices. An explicit expansion is given in [38]. It was shown in [43] that these discrete representations labeled by N appear naturally as a Fock space quantization of the 2D corner symplectic fluid.

此处 $\varepsilon_A{}^B$ 对应球面上一个复结构的选取，即它满足 $\varepsilon_A{}^B \varepsilon_B{}^C = -\delta_A{}^C$ 。我们可以看到，LQG 角对称群的一般离散表示给每个刺点分配了三个量子数 (j, Δ, s) 。与 (j, Δ, s) 关联的态可以展开为仅由 SU(2) 指标标记的自旋网络态的叠加，文献 [38] 中给出了具体展开形式。文献 [43] 已证明，这些由 N 标记的离散表示自然成为二维角辛流形的福克空间量子化结果。

These preliminary considerations should give ample motivations to explore the possibility of representing the corner symmetry group and connecting the ensuring representations to an extension of the quantum geometric picture arising from LQG, including in particular diffeomorphism symmetry. An important point of clarification is that, although spin foam models do not represent diffeomorphism symmetry, canonical LQG does by simply allowing the embedded graph to be displaced by a diffeomorphism. What we can now clearly see from (21) is that the LQG representation of diffeomorphism is trivial, i.e., it corresponds to choosing $(\Delta, s) = (0, 0)$.

上述初步讨论已充分说明，研究角对称群的表示并将所得表示与 LQG 量子几何图景的推广 (尤其是包含微分同胚对称性的推广) 相联系是很有必要的。需要明确的一点是: 虽然自旋泡沫模型不表示微分同胚对称性，但正则 LQG 通过允许嵌入图被微分同胚位移实现了该对称性。我们现在可以从式 (21) 中清楚看到，LQG 的微分同胚表示是平凡的，即它对应选取 $(\Delta, s) = (0, 0)$ 。

Boundaries in Loop Quantum Gravity

圈量子引力中的边界

We have seen in the previous section that corner symmetries allow us to understand the entanglement of quantum spacetime from the representation theory of corner symmetry groups. From the perspective of the corner symmetry groups, the fundamental quantum discreteness of geometry is a simple consequence of representation theory. These results suggest a form of local holography, where the boundary data for local subregions plays a pivotal role in the construction of the physical quantum states.

我们在前一节已经看到，角对称性允许我们从角对称群的表示理论出发理解量子时空的纠缠。从角对称群的视角来看，几何的基本量子离散性是表示理论的一个直接推论。这些结果引出了一种形式的局域全息论，其中局域子区域的边界数据在物理量子态的构建中发挥关键作用。

These ideas are profoundly connected to earlier LQG insights, for example, through the work on isolated horizons and black hole entropy [44], on the relationship between holography and quantum entanglement [45], and on the geometrical understanding of spin networks and spin foams (e.g., through twisted geometries) [46-48]. One element that the new corner perspective brings however is the seamless connection with the continuum QFT and the importance of diffeomorphism symmetry and its representation labels such as (Δ, s) . It also opens up the possibility to connect with the vast literature on holographic approaches to quantum gravity and provides new avenues of development for the core LQG results such as a Lorentz covariant formulation of spin network states with discrete area spectra.

这些思想与圈量子引力 (LQG) 此前的研究洞见有着深刻联系，例如体现在关于孤立视界与黑洞熵 [44]、全息论与量子纠缠的关系 [45]，以及对自旋网络与自旋泡沫的几何理解 (比如通过扭几何)[46-48] 的工作中。不过，新的角视角带来的一点优势，是它实现了与连续量子场论的无缝衔接，并且凸显了微分同胚对称性及其表示标签 (例如 (Δ, s)) 的重要性。它也为连接量子引力全息方法的大量研究成果提供了可能，还为 LQG 核心结论的发展开辟了新路径，比如得到了具有离散面积谱的洛伦兹协变自旋网络态表述。

We now briefly summarize some results that highlight the role played by boundaries in LQG. We refer the reader to the other review chapters [44] for black hole entropy, [45] for entanglement in LQG, [2] for further details on quantum geometry and spin networks, and [49] for spin foams and renormalization.

现在我们简要总结一些突出边界在 LQG 中作用的研究成果。关于黑洞熵请读者参考其他综述章节 [44]，关于 LQG 中的纠缠参考 [45]，关于量子几何与自旋网络的更多细节参考 [2]，关于自旋泡沫与重整化参考 [49]。

Black hole entropy in LQG One of the key ideas of LQG concerning black holes was proposed by Krasnov [50], building up on the previous work by Smolin [51] about edge mode degrees of freedom in gravity and TQFTs. The key idea first developed was that black hole entropy really counts the numbers of quantum degrees of freedom that live on the black hole horizon and represent quantum geometrical states [52,53]. Early on, a connection between these quantum states and horizon edge modes in the gravity phase space was conjectured [54]. These results led on the classical side to the study of boundary conditions for black holes and the proposal of isolated horizons [55-57]. It also led to a refined way of counting black hole microstates [58-60] and eventually to important realization of the connection with U(1) and SU(2) Chern-Simons theory living on the horizon [61-64]. In these works, the central role of the boundary symplectic structure was put forward, and it was finally understood that horizon punctures could also support loop algebra symmetries [65]. These results from the black hole entropy counting led to a direct connection with the corner symmetry in [66]. Note also that the notion of isolated horizons was recently generalized to the notion of non-expanding horizons [67]. These horizons correspond to null surfaces where the corner symmetry charges are conserved, and the corresponding Carrollian fluid is perfect, see section "Carrollian Fluid Conservation Law."

LQG 中的黑洞熵 LQG 关于黑洞的核心思想之一由 Krasnov 基于 Smolin 先前关于引力和拓扑量子场论中边缘模自由度的工作 [51] 提出 [50]。最初形成的核心观点是: 黑洞熵实际统计的是栖息在黑洞视界上、代表量子几何态的量子自由度的数量 [52,53]。早年研究就猜想这些量子态与引力相空间中的视界边缘模存在关联 [54]。这些结果推动了经典层面上对黑洞边界条件的研究，并催生了孤立视界的概念 [55-57]。它也带来了更精细的黑洞微态计数方法 [58-60]，最终让人们重要地认识到，这些量子态与存活在视界上的 U(1) 和 SU(2) 陈-西蒙斯理论存在联系 [61-64]。在这些工作中，边界辛结构的核心地位被提出，人们最终理解到，视界穿刺也可以承载圈代数对称性 [65]。这些来自黑洞熵计数的结果在文献 [66] 中被建立了与角对称性的直接关联。另外值得注意，孤立视界的概念最近被推广为非膨胀视界 [67]。这类视界对应零曲面，角对称荷在零曲面上守恒，对应的卡罗尔流体是理想流体，参见“卡罗尔流体守恒定律”小节。

LQG and twisted geometries We have seen that the entangling sphere carries representations of the boundary symmetries. The traditional loop gravity picture assigns, on the other hand, geometrical data to the vertex inside the sphere. So there seems to be tension between the two interpretations. This tension is resolved by the twisted geometry interpretation of spin networks, developed in [46,68]. In twisted geometries, one assigns geometrical elements to each link that intertwines two vertices and then proves that the matching of representations across the link allows one to reconstruct the connection. In a twisted geometry, there are two discrete geometries across the link: the one from the left and the one from the right. If one tries to identify the group data in terms of polyhedral geometries, one finds a mismatch: while the areas are matched, the shapes of the two adjacent polyhedra might differ; hence the discrete geometry is twisted. Remarkably, it was shown in [69] that the difference in shapes can be encoded into an SL(2, R) connection. In [70] it was also shown, following [71], that one has a natural generalization of spin network states that also carries representations of $SL(2,\mathbb{R})$ necessary to reconstruct the frame field. It was proven in [33] that the generator of this $SL(2,\mathbb{R})$ symmetry is, in the continuum, the induced geometry of the corner sphere transverse to the spin network link. Finally, in [72] the change of this $SL(2,\mathbb{R})$ was shown to encode radiation.

LQG 与扭几何我们已经知道，纠缠球面承载着边界对称性的表示。而传统圈引力的图像是将几何数据赋予球面内部的顶点。因此两种解释之间似乎存在矛盾。文献 [46,68] 提出的自旋网络扭几何诠释解决了这一矛盾。在扭几何中，人们将几何元素分配给连接两个顶点的每一条链，然后证明，链上表示的匹配可以重构联络。在扭几何中，一条链的两侧分别对应两个离散几何: 左侧几何和右侧几何。如果尝试用多面体几何来标识群数据，会发现不匹配: 面积是匹配的，但两个相邻多面体的形状可能不同；因此离散几何是“扭曲”的。值得注意的是，文献 [69] 已证明，形状差异可以编码进一个 SL(2, R) 联络。文献 [70] 在 [71] 的基础上还证明，存在自旋网络态的自然推广，这类态也承载重构标架场所必需的 $SL(2,\mathbb{R})$ 表示。文献 [33] 已证明，在连续体中，这个 $SL(2,\mathbb{R})$ 对称性的生成元就是垂直于自旋网络链的角球面的诱导几何。最后，文献 [72] 证明，$SL(2,\mathbb{R})$ 的变换可以编码辐射。

Demanding that the geometry is not twisted projects twisted geometries, which represent LQG states, onto Regge geometries [73]. The fact that LQG geometries are twisted and differ from the Regge geometries creates a puzzle. This puzzle can be simply resolved if one accepts that the faces dual to loop gravity states are not planar, and the edges of the dual polyhedra are spinning instead of being straight [74]. Moreover, we know from [48] that there is an isomorphism between twisted geometries representing LQG states and piecewise flat geometries which provides an exact continuum description of the LQG phase space in agreement with the results described in this review. This result also gives a geometrical understanding of the dual LQG vacua constructed in [75-77]. Another prescient paper by Bianchi [47] proposed early on a similar understanding of classical LQG geometry compatible with the action of diffeomorphisms. Now is a good time to go back to these results and use them to connect LQG and corner symmetries more tightly.

要求几何非扭将代表 LQG 态的扭几何投影到里奇几何 [73]。LQG 几何为扭且区别于里奇几何这一事实引出了一个谜题。只要我们承认圈引力态对偶的面不是平面的，且对偶多面体的边是旋转而非直的，这个谜题就能被简单解决 [74]。此外，我们从文献 [48] 得知，代表 LQG 态的扭几何与分段平坦几何之间存在同构，这为 LQG 相空间提供了符合本综述所述结果的精确连续描述。该结果也让我们从几何角度理解了文献 [75-77] 中构造的对偶 LQG 真空。比安基的另一篇前瞻性论文 [47] 很早就提出了对经典 LQG 几何的类似理解，该理解与微分同胚的作用相容。现在正是回到这些结果，并用它们更紧密地连接 LQG 和角对称性的好时机。

Entanglement in LQG, spin foams, and group field theories Very early on it was proposed that the area entanglement and its relation to boost symmetry could be recovered from the spin network entanglement of links carrying $\mathrm{SL}(2,\mathbb{C})$ representations [78, 79] . Since then there has been a recent interest in computing entanglement entropy of spin network states and trying to relate it to the Ryu-Takayanagi formula [80, 81]. The central idea here is that quantum geometry arises from a network of entanglement. The quantum entanglement in spin networks is encoded through the representation link [24, 82-84]. Similar ideas about entanglement have been explored in GFTs [80]. These results are a discrete expression of the fundamental ideas presented earlier, namely that entanglement can be derived from corner symmetry. It would be really interesting to develop further the connection between entropy counting and the renewed understanding of symmetry. In particular, in order to connect to the Unruh calculation, one would need to construct, in quantum gravity, the spacetime boost operator. Again, we refer the reader to the chapter [45] for more details about entanglement in LQG.

LQG、自旋泡沫和群场论中的纠缠很早就有人提出，面积纠缠及其与快度对称性的关系可以从携带 $\mathrm{SL}(2,\mathbb{C})$ 表示 [78, 79] 的连线的自旋网络纠缠中得到。此后近来，人们一直对计算自旋网络态的纠缠熵并尝试将其与 Ryu-Takayanagi 公式联系起来产生了兴趣 [80, 81]。此处的核心思想是量子几何产生于纠缠网络。自旋网络中的量子纠缠通过表示连线编码 [24, 82-84]。关于纠缠的类似思想已经在群场论中得到研究 [80]。这些结果是前文提出的基本思想——即纠缠可以从角对称性导出——的离散表达。进一步发展熵计数与更新后的对称性理解之间的联系会非常有意义。特别是，为了连接安鲁计算，我们需要在量子引力中构造时空快度算符。另外，关于 LQG 中纠缠的更多细节，我们建议读者参考章节 [45]。

Spin foams as quantization of boundary data Spin foams are based on a lattice truncation. The basic idea is to glue flat building blocks in such a way that Einstein's equations are satisfied at large scales compared to the typical lattice scale. The resulting spin foam amplitudes are a sum over geometric data in each fundamental four-simplex [49, 85, 86] . Each four-simplex is flat and contains no radiative data inside. It is only through the non-trivial gluing between adjacent four-simplices that curved geometries arise. Curvature is distributional and concentrated at surfaces dual to the fundamental triangles. In fact, it goes deeper than this: the traditional formulation of spin foam models focuses on simplicity constraints at the vertices of the spin foam [87,88], thereby restricting the geometry of boundary tetrahedra. One can take an equivalent point of view where the simplicity constraints are imposed by selecting appropriately the propagator going between the spin foam vertices, hence by controlling the corner geometry matching. This point of view adapted to corner and twisted geometry is already present in [88], was further developed in [89], and shown to be extremely efficient in the proof of spin foam asymptotics [90]. This twisted geometry perspective on spin foams was first discovered in the context of GFT models for quantum gravity [91,92] where one witnessed that it is possible to encode quantum gravity constraints by the right choice of propagator while keeping the interaction vertex simple.

作为边界数据量子化的自旋泡沫自旋泡沫基于格点截断。其基本思想是将平坦构造块粘合在一起，使得在远大于典型格点尺度的大尺度上满足爱因斯坦方程。由此得到的自旋泡沫振幅是对每个基本四单形 [49, 85, 86] 中几何数据的求和。每个四单形都是平坦的，内部不包含辐射数据。弯曲几何只能通过相邻四单形之间的非平凡粘合产生。曲率是分布性的，集中在基本三角形对偶的曲面上。实际上，这一点还有更深层的含义: 传统的自旋泡沫模型表述聚焦于自旋泡沫顶点处的简单性约束 [87,88]，从而限制了边界四面体的几何。我们可以采取一个等价观点: 通过适当选择连接自旋泡沫顶点的传播子施加简单性约束，也就是通过控制角几何匹配实现。这种适配角和扭几何的观点已经出现在文献 [88] 中，后在文献 [89] 得到进一步发展，并且在自旋泡沫渐近的证明中被证明是极为高效的 [90]。这种自旋泡沫的扭几何视角最初是在量子引力的 GFT 模型背景下发现的 [91,92]，在该背景下人们发现，可以通过正确选择传播子来编码量子引力约束，同时保持相互作用顶点简单。

What is missing? In asymptotically flat spacetimes, the simplest Dirac observables are the ADM energy and momentum, angular momentum, and center of mass together with an infinite-dimensional algebra of corner charges. The same happens at a finite distance. There are infinitely many corner charges that are Dirac observ-ables on the physical phase space of a bounded region. These charges generate corner symmetries such as super-rotations (which represent tangent diffeomor-phisms), super-boosts, and super-translations. We have seen in section ”Quantum Geometry and Corner Symmetry” that, even when we are in a vacuum state, some of the charges such as the corner boost charges do not vanish. Therefore, a generic physical state should carry a non-trivial representation of the corner symmetries. The problem with the usual spin network representation is that the basis states do not carry the corresponding representation labels. We know that these labels characterize components of the 4D metric at the corner. For instance, the generator of Diff(S) symmetry knows about the off-diagonal components of the metric in metric gravity [5] or knows about the torsion of the Ashtekar-Barbero connection in loop gravity [38, 66, 71] . Another example concerns the geometry symmetry encoded into an $SL(2,\mathbb{R})$ edge group, as described in the next section. Its representation labels describe the quantization of the tangential metric component [33,71]. Importantly, we lack a proper representation of asymptotically flat spacetimes in terms of spin network states (see [93]), which carries a representation of BMS symmetry. By connecting the boundary charges in a finite region to asymptotic charges at infinity, we obtain a new perspective to this long-standing problem in the field (see [4, 94, 95] for this connection at the semi-classical level).

缺失的是什么？在渐近平直时空中，最简单的狄拉克可观测量是 ADM 能量、动量、角动量和质心，再加上角电荷的无限维代数。有限距离处的情况也是如此。在有界区域的物理相空间上，存在无穷多个作为狄拉克可观测量的角电荷。这些电荷生成角对称性，例如超旋转 (代表切微分同胚)、超 boost 和超平移。我们在“量子几何与角对称性”一节中已经看到，即使处于真空态，部分电荷例如角 boost 电荷也不会消失。因此，一般的物理态应当携带非平凡的角对称性表示。通常自旋网络表示的问题在于，基态不携带对应的表示标签。我们知道这些标签刻画了角处四维度量的分量。例如，Diff(S) 对称性的生成元对应度量引力中度量的非对角分量 [5]，或是圈引力中阿希特卡-巴贝罗联络的挠率 [38, 66, 71] 。另一个例子是编码在 SL (2, ℝ) 边群中的几何对称性，我们会在下一节介绍。它的表示标签描述了切向度量分量的量子化 [33,71]。重要的是，我们目前缺少用自旋网络态描述的、携带 BMS 对称性表示的渐近平直时空的恰当表示 (参见 [93])。将有限区域的边界电荷与无穷远的渐近电荷联系起来，我们就能为该领域这个长期存在的问题提供全新视角 (半经典层面的该联系参见 [4, 94, 95])。

By not representing these symmetries at the quantum level, we lose information about the quantum geometry. In LQG, we have quantum numbers for internal SU(2) frame rotations and corresponding intertwiners, but there are no representation labels for the rest of the corner symmetry group. For instance, as shown in [66], with SU (2) labels, we can represent the flux but miss SL (2, ℝ) labels needed to represent the frame.

如果不在量子层面表示这些对称性, 我们就会丢失量子几何的信息。在 LQG 中, 我们已有内部 SU(2) 框架旋转的量子数和对应的交错算符，但角对称性群其余部分没有对应的表示标签。例如正如文献 [66] 所示，有了 SU (2) 标签，我们可以表示通量，但缺少表示框架所需的 SL (2, ℝ) 标签。

In [66], we have shown preliminary results on how to include the conformal weights and spin labels of Diff (S) representations, see (21), in the spin network description. The main result is that these edge labels control the gluing of states when we glue two elementary quantum subregions along an edge. It forces specific spin network superpositions to enter, such as $|\Delta, s\rangle\langle\Delta, s| = \sum_{j\geq s} \rho_j(\Delta, s) \ \ |j, m\rangle\langle j, m|$. These superpositions arise from the matching of diffeomorphism symmetry which determines the weights $\rho_j(\Delta, s)$. Therefore, having extra representation labels does not necessarily mean that spin networks are a wrong basis. It means that no physical states can be supported on a single spin network state. Since the nature of quantum geometry entanglement cannot simply be described by sharing an SU(2) spin. One needs extra symmetry labels control what superposition of states is allowed when we glue two quantum subregions together. In other words, the quantum geometry entanglement of subregions is considerably reinforced by the presence of diffeomorphism symmetry at the quantum level (As we have seen in section ”Representation Theory of Corner Symmetries,” the usual formulation of LQG contains only trivial representations of the diffeomorphism group that do not create any new entanglement.). Ultimately this is what we want: the demand of invariance under super-rotation, super-boost, and super-translation should constrain the admissible spin network superposition allowed in the edge gluing by requiring that these form representation states of the corner symmetry group. The fact that after gluing the representation labels are matched is the expression that the Hamiltonian and diffeomorphism constraints are implemented. So it should be now clear that not including these representation labels in the construction of spin network states in LQG is problematic. It is therefore of utmost importance to revisit the investigations of quantum geometry entanglement described in [45] in light of these results.

在文献 [66] 中，我们给出了如何在自旋网络描述中纳入 Diff(S) 表示的共形权与自旋标记的初步结果，见式 (21)。主要结论是，当我们沿一条边粘合两个基本量子子区域时，这些边标记控制着态的粘合。它迫使我们引入特定的自旋网络叠加，例如 $|\Delta, s\rangle\langle\Delta, s| = \sum_{j\geq s} \rho_j(\Delta, s)\ |j, m\rangle\langle j, m|$ 。这些叠加产生于微分同胚对称性的匹配，而微分同胚对称性决定了权 $\rho_j(\Delta, s)$ 。因此，额外的表示标记并不意味着自旋网络是错误的基，而是说明单个自旋网络态上无法承载任何物理态。这是因为量子几何纠缠的本质不能简单地通过共享一个 SU(2) 自旋来描述。当我们粘合两个量子子区域时，需要额外的对称标记来控制允许什么样的态叠加。换言之，量子水平上微分同胚对称性的存在极大地增强了子区域的量子几何纠缠 (正如我们在“角对称性的表示论”一节中看到的，圈量子引力的通常公式化仅包含微分同胚群的平凡表示，这类表示不会产生任何新的纠缠)。这最终正是我们想要的: 对超旋转、超 boost 和超平移下不变性的要求，应当通过要求这些态构成角对称群的表示态，来限制边粘合过程中允许的可接受自旋网络叠加。粘合后表示标记相互匹配这一事实，正是哈密顿约束与微分同胚约束得到实现的体现。因此现在可以清楚看出，在圈量子引力中构造自旋网络态时不包含这些表示标记是存在问题的。因此，根据这些结果重新探究文献 [45] 中描述的量子几何纠缠研究至关重要。

For the spin foam models, we can still use boundary states which are spin networks. What the corner symmetry analysis provides, however, is the possibility to constrain the value of spin foam amplitudes via symmetries. It is well known that in the presence of symmetries, path integral amplitudes satisfy Ward identities which express the invariance of the quantum amplitude under a symmetry transformation of the boundary state. We should therefore expect spin foam models and GFT models to satisfy such Ward identities. These identities represent the quantization of the infinite set of flux-balance laws that arise from solving the Hamilton and vector constraints. At asymptotic infinity, these balance laws turn into Ward identities, called soft theorems, that put stringent constraints on the perturbative S-matrix [19]. This means that an exciting new possibility for spin foam models is opening up: the possibility to identify the quasi-local generators of symmetry charges acting on the spin network states and write down the Ward identities that spin foam models ought to satisfy.

对于自旋泡沫模型，我们仍然可以使用边界态，也就是自旋网络。但角对称性分析的作用在于，它提供了通过对称性约束自旋泡沫振幅取值的可能。众所周知，当存在对称性时，路径积分振幅满足沃德恒等式，该恒等式体现了量子振幅在边界态的对称变换下的不变性。因此我们应当期望自旋泡沫模型和群场论 (GFT) 模型满足这类沃德恒等式。这些恒等式代表了通过求解哈密顿约束和矢量约束得到的无穷多通量平衡定律的量子化。在渐近无穷远处，这些平衡定律变为沃德恒等式，也称为软定理，它对微扰 S 矩阵给出严格约束 [19]。这意味着自旋泡沫模型迎来了一个令人振奋的新可能: 我们有可能确定作用在自旋网络态上的准局域对称荷生成元，并写下自旋泡沫模型应当满足的沃德恒等式。

Classical Theory and Symplectic Structure

经典理论与辛结构

We have already briefly discussed above in section ”Representation Theory of Corner Symmetries” how LQG representation of quantum geometry naturally arises from studying the corner symmetries group appearing in tetrad gravity with a gauge-fixed internal normal. The choice of this internal normal breaks Lorentz invariance and determines an internal SU(2) group. Our goal is now to show how a generalization of the LQG representation of quantum geometry, covariant under the internal Lorentz group, naturally arises from studying the corner symmetries of tetrad gravity. This result follows from a detailed canonical analysis of the first-order Lagrangian for gravity.

我们已经在上文“边界对称性的表示论”一节中简要讨论过，研究带规范固定内法向的标架引力中出现的边界对称群时，圈量子引力 (LQG) 的量子几何表示会自然涌现。选择该内法向会破坏洛伦兹不变性，并确定一个内 SU(2) 群。我们现在的目标是展示，如何从研究标架引力的边界对称性，自然得到洛伦兹群协变的推广版 LQG 量子几何表示。该结果源于对一阶引力拉格朗日量的详细正则分析。

Let us consider a tetrad 1-form e^I and a connection 1-form ω^{IJ} with curvature F^{IJ} . These fields can be used to build the Lagrangian 4-form for Einstein-Cartan-Holst gravity

我们考虑一个标架 1-形式 e^I 和一个带曲率 F^{IJ} 的联络 1-形式 ω^{IJ} 。可以用这些场构造爱因斯坦-卡坦-霍尔斯特引力的拉格朗日 4-形式

$$L_{\mathrm{ECH}} = \frac{1}{2} E_{IJ} \wedge F^{IJ}, \ E_{IJ}[e] = (\star + \beta)(e_I \wedge e_J). \tag{22}$$

We are going to focus primarily on the symplectic structure of this Lagrangian. In its analysis, a crucial role is played by the internal normal $n^I = n^\mu e^I_\mu$ obtained from the time-like normal n^μ to a space-like hypersurface $\sum$. This normal is such that $n^I n_I = n^\mu n_\mu = -1$. The importance of keeping track of this normal in the phase space was already recognized in various contexts in [63,79,96-101].

我们将主要关注该拉格朗日量的辛结构。在分析过程中，由类空超曲面 $\sum$ 的类时法向 n^μ 得到的内法向 $n^I = n^\mu e^I_\mu$ 起到关键作用。该法向满足 $n^I n_I = n^\mu n_\mu = -1$ 。相空间中需要跟踪该法向的重要性，早已在 [63,79,96-101] 的不同研究场景中得到认可。

Using the internal normal, we can decompose the Lorentz connection as

利用内法向，我们可以将洛伦兹联络分解为:

$$\omega^{IJ} = \Gamma^{IJ} - 2K^{[I}n^{J]}, \ \mathrm{d}_\Gamma n^I = 0, \ \ \mathrm{d}_\omega n^I = K^I. \tag{23}$$

We also decompose the pullback of the tensorial 2-form E^{IJ} as

我们还将张量 2-形式 E^{IJ} 的拉回分解为:

$$E^{IJ}\big|_{\sum} = -2E^{[I}n^{J]} + \beta\left(e^I \wedge e^J\right). \tag{24}$$

As explained at length in [33], with this decomposition the symplectic structure of the ECH Lagrangian becomes

正如 [33] 中详细解释的，通过该分解，爱因斯坦-卡坦-霍尔斯特 (ECH) 拉格朗日量的辛结构变为:

$$\Omega_{\text{ECH}} = \int_{\Sigma} \delta K^I \wedge \delta E_I + \oint_S \left(\delta E_I \delta n^I - \frac{\beta}{2} \delta e_I \wedge \delta e^I \right). \tag{25}$$

One can see that the normal appears in the boundary contribution and also that the frame is conjugated to itself on the boundary due to the non-vanishing Barbero-Immirzi parameter. To interpret this result, one should notice that the bulk term is nothing but the symplectic structure of canonical gravity. Indeed, with a slight rewriting, one can show that [33]

可以看到，法向出现在边界项中，并且由于巴贝罗-伊米尔齐参数非零，标架在边界上自共轭。为了解释该结果，我们需要注意，体项正是正则引力的辛结构。事实上，稍加改写就可以证明 [33]:

$$\int_{\Sigma} \delta K^I \wedge \delta E_I = \int_{\Sigma} \delta (K \times e)^I \wedge \delta e_I = \frac{1}{2} \int_{\Sigma} \delta p^{ab} \delta h_{ab} = \Omega_{\text{GR}}, \tag{26}$$

where $p^{ab} = \sqrt{h} \left(K h^{ab} - K^{ab} \right)$ is the usual momentum density of the ADM formulation. In summary, we can write (25) in the form

其中 $p^{ab} = \sqrt{h} \left(K h^{ab} - K^{ab} \right)$ 是 ADM 表述中常规的动量密度。综上，我们可以将 (25) 写为如下形式:

$$\Omega_{\text{ECH}} = \Omega_{\text{GR}} + \Omega^S_{\text{ECH/GR}}, \tag{27}$$

where $\Omega^S_{\text{ECH/GR}}$, which is the boundary contribution in (25), is the relative boundary symplectic structure between the ECH and canonical formulations of gravity. This is the main message of [3], namely that any formulation L_{F} of gravity has a symplectic structure which is of the form $\Omega_{\text{F}} = \Omega_{\text{GR}} + \Omega^S_{\text{F/GR}}$. The bulk symplectic structure is universal and encodes the fact that the theories being discussed are general relativity, while the boundary contribution is formulation dependent. Even if ECH gravity (22) is equivalent in the bulk to general relativity, it carries a specific boundary symplectic structure.

其中 $\Omega^S_{\text{ECH/GR}}$ 是 (25) 中的边界贡献，是 ECH 引力表述和正则引力表述之间的相对边界辛结构。这就是文献 [3] 的核心观点，即任意引力表述 L_{F} 的辛结构都形如 $\Omega_{\text{F}} = \Omega_{\text{GR}} + \Omega^S_{\text{F/GR}}$。体辛结构具有普适性，体现了所讨论理论都是广义相对论这一核心特征，而边界贡献则和具体表述相关。即使 ECH 引力 (22) 在体区域等价于广义相对论，它也拥有独特的边界辛结构。

Our goal is to understand the physical significance of the boundary symplectic structure (25). This latter is responsible for the appearance of non-vanishing charges for Lorentz transformations, and, as we will explain shortly, for the non-commutativity of the tangential boundary metric (which in turn leads to the discreteness of the boundary area). Note that studies of the relation between the Einstein-Hilbert and Einstein-Cartan-Holst formulations through a boundary term in the symplectic structure go back to [102]. The emphasis on relative charges was made more explicit in [10, 103].

我们的目标是理解边界辛结构 (25) 的物理意义。该结构是洛伦兹变换存在非零荷的原因，并且我们很快会说明，它也导致切向边界度量不对易，而这进一步带来边界面积的离散性。值得注意，通过辛结构中的边界项研究爱因斯坦-希尔伯特表述和爱因斯坦-卡坦-霍尔斯特表述之间关系的工作可以追溯到文献 [102]。对相对荷的强调在文献 [10, 103] 中得到了更明确的体现。

Charges and Discreteness

荷与离散性

First, let us stress that the treatment presented here is in a sense orthogonal to the usual approach to canonical LQG. Indeed, in standard treatments of LQG, one focuses on bulk variables, and in particular the contribution of the Holst term proportional to β is used to build a bulk connection variable. This is made possible by the identity $\beta \mathrm{d}\left(e_I \wedge \delta e^I\right) \doteq \beta\left(e_I \wedge e_J\right) \wedge \delta \omega^{IJ}[e]$, which holds on-shell of the torsion equation of motion. In the time gauge where $n^I = \delta_0^I$, this reduces to $\beta \mathrm{d}\left(e_i \wedge \delta e^i\right) \triangleq \beta E_i^a \delta \Gamma_a^i [e]$, where i is the index in the $\mathfrak{su}(2)$ subalgebra of $\mathfrak{so}(3,1)$ which survives the fixing of the normal. This is the identity that enables, once the Holst term is included, to define a bulk Ashtekar-Barbero connection variable conjugated to the electric field E . Here we wish to shift emphasis from the bulk to the boundary and therefore keep the contribution of the Holst term in the boundary symplectic structure.

首先，我们要强调，本文给出的处理在某种意义上与正则圈量子引力 (LQG) 的常用方法相互独立。实际上，在 LQG 的标准处理中，研究焦点是体变量，尤其是正比于 β 的霍尔斯特项贡献被用来构造体联络变量。这一点之所以成立，是因为恒等式 $\beta \mathrm{d}\left(e_I \wedge \delta e^I\right) \doteq \beta\left(e_I \wedge e_J\right) \wedge \delta \omega^{IJ}[e]$ 在挠率运动方程的壳上成立。在时间规范 $n^I = \delta_0^I$ 下，该恒等式约化为 $\beta \mathrm{d}\left(e_i \wedge \delta e^i\right) \triangleq \beta E_i^a \delta \Gamma_a^i [e]$ ，其中 i 是 $\mathfrak{so}(3,1)$ 的 $\mathfrak{su}(2)$ 子代数中的指标，该子代数在固定法向后保留。正是这个恒等式使得在引入霍尔斯特项后，能够定义与电场 E 共轭的体阿斯 ht 卡-巴贝罗联络变量。本文我们希望将研究重点从体转移到边界，因此保留霍尔斯特项在边界辛结构中的贡献。

The fact that the symplectic structure of ECH gravity differs from that of canonical gravity Ω_{GR} by the presence of a surface term has the important consequence that it gives rise to a new set of surface charges. These are the charges of the internal Lorentz transformations acting as $\delta_\alpha V^I = -\alpha^I{}_J V^J$ for $V^I = \left(E^I, n^I, e^I\right)$ and $\delta_\alpha \omega^{IJ} = \mathrm{d}_\omega \alpha^{IJ}$. These charges are obtained as usual by computing the contraction $I_\alpha \Omega$, and their explicit expression is

ECH 引力的辛结构与正则引力的辛结构 Ω_{GR} 不同，区别在于存在一个表面项，这一性质的重要结论是它会产生一组新的表面荷。这些就是内部洛伦兹变换的荷，变换作用为 $\delta_\alpha V^I = -\alpha^I{}_J V^J$ ，对应 $V^I = \left(E^I, n^I, e^I\right)$ 和 $\delta_\alpha \omega^{IJ} = \mathrm{d}_\omega \alpha^{IJ}$ 。这些荷可以像通常那样通过计算缩并 $I_\alpha \Omega$ 得到，其显式表达式为

$$G(\alpha) = \int_\Sigma E^{IJ} \wedge \mathrm{d}_\omega \alpha_{IJ} \triangleq \oint_S \alpha_{IJ} E^{IJ} = \oint_S \alpha_{IJ}\left(\beta\left(e^I \wedge e^J\right) - 2E^I n^J\right).$$

(28)

These boundary charges are the Lorentz covariant version of the SU(2) charges (16). They have also been discussed extensively in the context of black hole entropy counting in [64]. It is important to stress out that these charges are purely relative charges, in the sense that they are produced entirely by the boundary symplectic structure in (25). This means that, as expected, metric gravity with the symplectic structure Ω_{GR} does not possess Lorentz charges. This is the key property which differentiates, already at the classical level, LQG from any metric-based approach: it possesses extra surface charges that give rise to the basic surface flux operators. We have here presented the covariant formulation with an arbitrary internal normal n^I , but in LQG the normal is fixed to δ_0^I and one recovers the SU(2) fluxes (16) whose action on states is (19).

这些边界荷是 SU (2) 荷 (16) 的洛伦兹协变形式。它们也已经在文献 [64] 的黑洞熵计数背景下被广泛讨论。需要强调的是，这些荷纯粹是相对荷，它们完全由式 (25) 中的边界辛结构产生。这意味着，正如预期，带有辛结构 Ω_{GR} 的度规引力不存在洛伦兹荷。这就是在经典层面就已经区分 LQG 和任何基于度规的方法的关键性质:LQG 拥有额外的表面荷，这些额外表面荷衍生出了基本的表面通量算符。我们在这里给出了任意内部法向 n^I 下的协变表述，但在 LQG 中法向被固定为 δ_0^I，此时我们可以重新得到 SU(2) 通量 (16)，其在态上的作用为式 (19)。

We now turn to the main result obtained from the detailed analysis of the boundary symplectic structure, which is the discreteness of area implied by the presence of $\beta \neq 0$. In [33, 35] , we have decomposed the six components (There are six components because $e_a^I n_I = 0$, and on S we have $a = 1, 2$.) of the tangential frame e_a^I at the surface S into a so-called spin operator $S^I = \frac{1}{2}\beta(e \times e)^I$ and the tangential metric $q_{ab} = e_a^I e_b^J \eta_{IJ}$. The spin operator is the SU (2) flux E^i written in the internal Lorentz-covariant form. Equipped with this decomposition, one can show that the tangential metric satisfies the $\mathfrak{sl}(2, \mathbb{R})^S$ algebra

我们现在来看边界辛结构详细分析得到的主要结果，也就是 $\beta \neq 0$ 的存在暗示了面积的离散性。在 [33, 35] 中，我们分解了曲面 S 切框架 e_a^I 的六个分量 (存在六个分量是因为 $e_a^I n_I = 0$，且在 S 上我们有 $a = 1, 2$)，将其分解为所谓的自旋算符 $S^I = \frac{1}{2}\beta(e \times e)^I$ 和切度量 $q_{ab} = e_a^I e_b^J \eta_{IJ}$ 。自旋算符是写成内洛伦兹协变形式的 SU (2) 流 E^i 。基于这一分解，可以证明切度量满足 $\mathfrak{sl}(2, \mathbb{R})^S$ 代数

$$\{q_{ab}(x), q_{cd}(y)\} = -\frac{1}{\beta}\left(q_{ac}\varepsilon_{bd} + q_{bc}\varepsilon_{ad} + q_{ad}\varepsilon_{bc} + q_{bd}\varepsilon_{ac}\right)(x)\,\delta^2(x - y)\,. \tag{29}$$

This is in line with an earlier analysis carried out in [70,71] in the SU(2) gauge fixed formulation. This important result shows that the corner metric becomes noncommutative in the presence of the Barbero-Immirzi parameter. This shows the tangential metric component of S is the $\mathfrak{sl}(2, \mathbb{R})^S$ generators. One can then show that the quadratic Casimir is given by

这与早前在 SU(2) 规范固定表述中 [70,71] 完成的分析一致。这一重要结果表明，存在巴贝罗-伊米里齐参数时，角点度量具有非对易性。这说明 S 的切度量分量就是 $\mathfrak{sl}(2, \mathbb{R})^S$ 生成元。进而可以证明二次卡西米尔量由下式给出

$$C_{\mathfrak{sl}(2,\mathbb{R})^S} = \beta^2 \det(q)\,. \tag{30}$$

Since the metric is that of a two-dimensional space-like surface, the determinant on the right-hand side is positive, and we conclude from this relation that the $\mathfrak{sl}(2, \mathbb{R})^S$ Casimir is positive as well. This indicates that the surface area spectrum has to be labeled by the discrete series of representations, if we were to label states by unitary representations respecting the boundary symmetry structure.

由于该度量是二维类空曲面的度量，右侧的行列式为正，我们由此关系推出 $\mathfrak{sl}(2, \mathbb{R})^S$ 卡西米尔量也为正。这表明，如果我们按照尊重边界对称结构的幺正表示来标记态，那么表面积谱必须由离散表示序列标记

Note that in order to make this construction rigorous, a regularization of the corner algebra is necessary, as outlined in [70]. A treatment that also includes diffeomorphism symmetry on the sphere is also required, along the lines of [34, 36, 38, 43, 66] . In particular, in [43] a regularization in terms of spherical harmonics with respect to a fiducial metric at the sphere was introduced as an example of a regularization which does not involve any discretization. In [36] such a regularization was shown to be equivalent to a matrix model regularization and arising by rendering the corner sphere non-commutative. This is ultimately related to the understanding of quantization of diffeomorphisms in the corner symmetry group and the role of the dynamics. We therefore now turn to a description of the dynamics along null boundaries and show that in this context we can also obtain the discreteness of area from the canonical analysis of null surfaces.

请注意，为了让该构造严谨，正如 [70] 中概述的那样，必须对角点代数进行正则化，还需要按照 [34, 36, 38, 43, 66] 的思路，纳入球面的微分同胚对称处理。具体而言，[43] 中引入了一种基于球面基准度量的球谐函数正则化，作为不涉及任何离散化的正则化示例。[36] 中证明这种正则化等价于矩阵模型正则化，源于让角点球面成为非对易的。这最终关系到对角点对称群中微分同胚量子化以及动力学作用的理解。因此，我们接下来介绍类空边界上的动力学，并证明在此背景下，我们同样可以通过对类空曲面的正则分析得到面积离散性

Null Boundaries, Isolated Horizons, and Fluid Conservation Law

类光边界、孤立视界与流体守恒定律

In the previous sections, we have discussed the representation of the corner symmetry group which contains the symmetry generators with vanishing flux. In order to understand the dynamics, one needs to describe the representation of the extended corner symmetry group and include the super-translation generators that move the corner transversally. It has proven invaluable to focus on the representation of null super-translations and study the gravitational dynamics projected along null surfaces and null horizons. There has been a rich literature of the subject dating back to the membrane paradigm [104], the study of isolated horizons [54], and more recently the dynamics of null surfaces, the construction of the symplectic potential, of the symmetry charges, and their understanding in terms of Carrollian geometry [94, 105-112].

在之前的章节中，我们已经讨论了包含通量为零的对称性生成元的角对称性群的表示。为了理解动力学，我们需要描述扩展角对称性群的表示，并包含可横向移动角的超平移生成元。目前已经证实，聚焦于类空虚超平移的表示，研究沿类空曲面和类空视界投影的引力动力学是极有价值的。该主题已有大量研究成果，最早可追溯到膜范式 [104]、孤立视界研究 [54]，近年来则涵盖类空曲面动力学、辛势构造、对称性荷构造，以及基于卡罗尔几何对这些内容的理解 [94, 105-112]。

Carrollian Fluid Conservation Law

卡罗尔流体守恒定律

In this section we study the geometry and dynamics of a null boundary $\mathcal{N}$. On a null boundary, the pullback of the four-dimensional metric is degenerate. The degenerate direction is the direction of the null rays that generate the surface. This null direction is in the kernel of the induced metric

在本节中，我们研究类光边界 $\mathcal{N}$ 的几何与动力学。在类光边界上，四维度量的拉回是退化的，退化方向就是生成该曲面的类光射线的方向，这个类光方向属于诱导度量的核

$$q_{ab} = \varphi_{\mathcal{N}}^{*} g_{ab},\ \ell^a q_{ab} = 0. \tag{31}$$

Any such null direction is unique up to a local rescaling sending (We assume that ℓ^a is the future pointing.) ℓ^a into $e^{\lambda}\ell^a$. Mathematically this means that $\mathcal{N}$ is a fibered manifold with fibration $p : \mathcal{N} \to S$. The null direction ℓ is in the kernel of $\mathrm{d}p$ (push-forward of $p : \mathcal{N} \to S$), while the null metric q_{ab} can be understood as the pullback of a time-dependent metric q^S_{AB} on the base. The Lie derivative of the metric along ℓ decomposes in terms of the expansion and shear of the null surface:

任何这类类光方向在局部重标度下都是唯一的，重标度变换会将 ℓ^a 变为 $e^{\lambda}\ell^a$ (我们假设 ℓ^a 指向未来)。从数学上讲，这意味着 $\mathcal{N}$ 是带纤维化 $p : \mathcal{N} \to S$ 的纤维化流形。类光方向 ℓ 属于 $\mathrm{d}p$ ($p : \mathcal{N} \to S$ 的推前) 的核，而类光度量 q_{ab} 可以理解为基空间上依赖时间的度量 q^S_{AB} 的拉回。度量沿 ℓ 的李导数可以按类光曲面的膨胀和切变分解为:

$$\theta_{ab} := \frac{1}{2}\mathcal{L}_{\ell} q_{ab} = \frac{q_{ab}}{2}\theta + \sigma_{ab}. \tag{32}$$

Note that the degenerate metric q_{ab} determines a spatial area form ε_S on $\mathcal{N}$ such that $i_\ell \varepsilon_S = 0$. Moreover the choice of a null generator ℓ determines a volume form $\varepsilon_{\mathcal{N}}$ on $\mathcal{N}$ which is such that $i_\ell \varepsilon_{\mathcal{N}} = \varepsilon_S$. The expansion relates the two forms through the identity $\mathrm{d}\varepsilon_S = \theta \varepsilon_{\mathcal{N}}$.

请注意退化度量 q_{ab} 在 $\mathcal{N}$ 上确定了一个空间面积形式 ε_S ，满足 $i_\ell \varepsilon_S = 0$ 。此外，选择类光生成元 ℓ 会在 $\mathcal{N}$ 上确定一个体积形式 $\varepsilon_{\mathcal{N}}$ ，满足 $i_\ell \varepsilon_{\mathcal{N}} = \varepsilon_S$ 。膨胀通过恒等式 $\mathrm{d}\varepsilon_S = \theta \varepsilon_{\mathcal{N}}$ 关联这两种形式。

In order to construct a connection on $\mathcal{N}$ and describe the dynamics of null surfaces, it is convenient to introduce an Ehresmann connection. This is a 1- form k_a dual to the null vector ℓ^a : $k_a \ell^a = 1$ [113,114]. The Ehresmann connection defines a notion of horizontality, where Y is a horizontal vector field on $\mathcal{N}$ if $i_Y k = 0$. A general vector $\xi \in T\mathcal{N}$ can be decomposed as $\xi = T\ell + Y$, with Y horizontal. The Ehresmann connection also allows to decompose the volume form on $\mathcal{N}$ as $\varepsilon_{\mathcal{N}} = k \wedge \varepsilon_S$. The data $\mathcal{C}_{\mathcal{N}} = (q_{ab}, \ell^a, k_b)$ represents the Carrollian geometry of $\mathcal{N}$. This data defines a rigging structure [115], i.e., a projector $\Pi_a{}^b = q_a{}^b + k_a \ell^b$, where $q_a{}^b$ is the horizontal projector $\ell^a q_a{}^b = 0 = q_a{}^b k_b$. The presence of a rigging structure allows us to uniquely raise indices of horizontal tensors. For instance, given the expansion tensor θ_{ab} , the tensor $\theta_a{}^b$ is defined as the unique tensor such that $\theta_a{}^c q_{cb} = \theta_{ab}$ and such that $\theta_a{}^b k_b = 0$.

为了在 $\mathcal{N}$ 上构造联络并描述类光曲面的动力学，引入埃雷斯曼联络是很方便的。这是一个对偶于零向量 ℓ^a : $k_a \ell^a = 1$ 的 1-形式 k_a [113,114]。埃雷斯曼联络定义了水平性的概念: 若满足 $i_Y k = 0$ ，则 Y 是 $\mathcal{N}$ 上的水平向量场。任意向量 $\xi \in T\mathcal{N}$ 都可以分解为 $\xi = T\ell + Y$ ，其中 Y 是水平分量。埃雷斯曼联络还允许我们将 $\mathcal{N}$ 上的体积形式分解为 $\varepsilon_{\mathcal{N}} = k \wedge \varepsilon_S$ 。数据 $\mathcal{C}_{\mathcal{N}} = (q_{ab}, \ell^a, k_b)$ 描述了 $\mathcal{N}$ 的卡罗尔几何，该数据定义了一个索具结构 [115]，即一个投影算子 $\Pi_a{}^b = q_a{}^b + k_a \ell^b$ ，其中 $q_a{}^b$ 是水平投影算子 $\ell^a q_a{}^b = 0 = q_a{}^b k_b$ 。索具结构的存在允许我们唯一地升水平张量的指标。例如，给定膨胀张量 θ_{ab} ，张量 $\theta_a{}^b$ 被定义为满足 $\theta_a{}^c q_{cb} = \theta_{ab}$ 和 $\theta_a{}^b k_b = 0$ 的唯一张量。

The rigging structure allows us to introduce the notion of Carrollian connections D_a [110,112] . These are torsionless connections that preserve the rigging projector: $D_a \Pi_b{}^c = 0$. A Carrollian connection defines a 1-form ω_a on $\mathcal{N}$ given by the derivative of the volume form:

索具结构允许我们引入卡罗尔联络的概念 D_a [110, 112]。这是保持索具投影器的无挠联络: $D_a \Pi_b{}^c = 0$。卡罗尔联络在 $\mathcal{N}$ 上定义了一个 1-形式 ω_a，由体积形式的导数给出:

$$D_a \varepsilon_{\mathcal{N}} = -\omega_a \varepsilon_{\mathcal{N}}. \tag{33}$$

This 1-form can be decomposed into a transverse and horizontal component as $\omega_a = \kappa k_a + \pi_a$, where π_a is the Hajicek connection, while κ is the surface gravity measuring the inaffinity of $\ell.\omega_a$ can be understood as a boost connection since, under the boost rescaling $\ell \to e^{\lambda}\ell$ and $k \to e^{-\lambda}k$, it transforms as $\omega_a \to \omega_a + \partial_a \lambda$. One can then prove that the boost covariantized derivative of ℓ^b and k_b is horizontal. They define the expansion and transverse expansion tensors

该 1-形式可以分解为横向分量和水平分量，即 $\omega_a = \kappa k_a + \pi_a$，其中 π_a 是哈伊切克联络，而 κ 是衡量 $\ell.\omega_a$ 非仿射性的表面引力，可以理解为 boost 联络，因为在 boost 重标度 $\ell \to e^{\lambda}\ell$ 和 $k \to e^{-\lambda}k$ 下，它变换为 $\omega_a \to \omega_a + \partial_a \lambda$。进而可以证明，$\ell^b$ 和 k_b 的 boost 协变导数是水平的。它们定义了膨胀张量和横向膨胀张量

$$(D_a - \omega_a)\,\ell^b = \theta_a{}^b,\; q_a{}^c\,(D_c + \omega_c)\,k_b = \theta^{(k)}_{ab}. \tag{34}$$

The Carrollian connection preserves the metric when derivatives and tensor indices are taken to be transverse directions. In general, one has

当导数和张量指标都取横向方向时，卡罗尔联络保持度规。一般来说，我们有

$$D_a q_{bc} = -\left(k_b \theta_{ac} + k_c \theta_{ac}\right). \tag{35}$$

We therefore see that the Carrollian connection compatible with the Carroll structure $\left(q_{ab}, \ell^b, k_b\right)$ is determined by the boost 1-form and the transverse expansion $\left(\omega_a, \theta^{(k)}_{ab}\right)$. It is well known that the projection of Einstein's equation along the null surface leads to two sets of equations: the Raychaudhuri and Damour equations [116]. These are given by

因此我们可以看到，与卡罗尔结构 $\left(q_{ab}, \ell^b, k_b\right)$ 相容的卡罗尔联络由 boost 1-形式和横向膨胀 $\left(\omega_a, \theta^{(k)}_{ab}\right)$ 决定。众所周知，爱因斯坦方程沿类光曲面的投影给出两组方程: 雷乔德里方程和达穆尔方程 [116]。其形式为

$$(\ell + \theta)\,[\theta] = \mu\theta - \sigma_{ab}\sigma^{ab}, \tag{36}$$

$$q_a{}^b\,(\mathcal{L}_\ell + \theta)\,[\pi_b] = \bar{D}_a \mu - \bar{D}_b \sigma^{ab}, \tag{37}$$

where we have introduced the surface tension $\mu := \left(\kappa + \frac{\theta}{2}\right)$ of $\mathcal{N}$ and $\bar{D}_a = q_a{}^b D_b$ denotes the horizontal derivative.

其中我们引入了 $\mathcal{N}$ 的表面张力 $\mu := \left(\kappa + \frac{\theta}{2}\right)$，$\bar{D}_a = q_a{}^b D_b$ 表示水平导数。

As shown in [107], these equations can be understood as conservation equations of corner symmetry charges associated with super-translations along S and diffeomorphism along S . The corresponding charge aspects are $\theta\varepsilon_S$ for super-translations and $\pi_A\varepsilon_S$ for super-rotations. These two equations can be understood as conservation equations for Noether charges with non-trivial flux as in (4), where the symplectic potential (This expression is valid in the boost frame where $k_a\delta\ell^a = 0$.

正如文献 [107] 所示，这些方程可以理解为与超平移沿 S 、微分同胚沿 S 相关的角对称性荷的守恒方程。对应的荷方面，超平移为 $\theta\varepsilon_S$ ，超旋转为 $\pi_A\varepsilon_S$ 。这两个方程可以理解为具有非平凡通量的诺特荷的守恒方程，如式 (4) 所示，其中辛势 (该表达式在 $k_a\delta\ell^a = 0$ 的 boost 参考系中成立。

The boost transformation is pure gauge in the metric formulation of gravity, while it possesses a non-trivial charge in the first-order formulation as we are about to see.) on $\mathcal{N}$ is given by [11, 106, 117]

Boost 变换在引力的度规表述中是纯规范的，但正如我们即将看到的，它在一阶表述中具有非平凡荷。) 在 $\mathcal{N}$ 上的表达式为 [11, 106, 117]

$$\Theta_{\mathcal{N}} = \int_{\mathcal{N}} \left(\frac{1}{2}\left(\sigma^{ab} - \mu q^{ab}\right)\delta q_{ab} - \pi_a\delta\ell^a\right)\varepsilon_{\mathcal{N}}. \tag{38}$$

From this, we can read the canonical conjugate pairs of spin-0 $\left(\mu, \sqrt{q}\right)$, spin-1 (π_a, ℓ^a) , and spin-2 $\left(\sigma^{ab}, q_{ab}\right)$.

由此我们可以读出自旋 0 $\left(\mu, \sqrt{q}\right)$ 、自旋 1 (π_a, ℓ^a) 和自旋 2 $\left(\sigma^{ab}, q_{ab}\right)$ 的正则共轭对。

Quite remarkably, these equations are now understood as conservation equations for a Carrollian fluid [109,110,112]. The Carrollian fluid stress tensor is given in terms of the rigging structure and the Carrollian connection simply as

非常值得注意的是，这些方程现在被理解为卡罗尔流体的守恒方程 [109,110,112]。卡罗尔流体的能压张量可以通过索具结构和卡罗尔联络简单表示为

$$T_a^b \ := \ D_a\ell^b - \Pi_a{}^b D_c\ell^c. \tag{39}$$

The tensor $D_a\ell^b = \omega_a\ell^b + \theta_a{}^b$ is called the shape operator or Weingarten map. As first shown in [110] (see also [112]), the Einstein equations pulled back on $\mathcal{N}$ can then simply be written as fluid conservation equations

张量 $D_a\ell^b = \omega_a\ell^b + \theta_a{}^b$ 被称为形状算子或外尔加滕映射。正如文献 [110] 首次证明的 (亦见 [112]), 拉回至 $\mathcal{N}$ 的爱因斯坦方程可以简单写为流体守恒方程

$$\Pi_a{}^b G_{\ell b} = D_b T_a{}^b = 0. \tag{40}$$

This generalizes for null surfaces the celebrated Brow-York result [118]. In [112] it was shown that these equations can be obtained from the gravitational symplectic potential and that the symmetry charges associated with a vector field ξ tangent to N are

这将著名的 Brow-York 结果 [118] 推广到了类光曲面。文献 [112] 表明，这些方程可从引力辛势导出，且与切于 N 的矢量场 ξ 相关的对称荷为

$$Q(\xi) = \int_S \xi^a T_a^b \varepsilon_b, \tag{41}$$

where $\varepsilon_b = i_{\partial_b}\varepsilon_{\mathcal{N}}$. The Carrollian fluid energy-momentum tensor can be decomposed in terms of the quantities defined above as

其中 $\varepsilon_b = i_{\partial_b}\varepsilon_{\mathcal{N}}$ 。卡罗尔流体能量动量张量可以用上述定义的量分解为

$$T_a{}^b = -\theta k_a \ell^b + \pi_a \ell^b - \mu q_a{}^b + \sigma_a{}^b. \tag{42}$$

This energy-momentum tensor can be interpreted as that of a Carrollian fluid obtained from the $c \to 0$ limit of a relativistic energy-momentum tensor. In this fluid analogy, θ plays the role of the fluid energy, π_a that of the heat flux, $-\mu$ of the pressure, and σ_{ab} of the viscous stress tensor. We therefore see that it is not only the representation theory of the corner symmetry group that leads to a two-dimensional fluid analogy but also the gravitational dynamics along null surfaces in agreement with the membrane paradigm [104]. The notion of non-expanding horizons [67,119] can now be simply understood in this modern language as a perfect fluid where the energy θ and the dissipation tensor σ_{ab} vanish.

该能量动量张量可以解释为相对论能量动量张量取 $c \to 0$ 极限得到的卡罗尔流体的能量动量张量。在这个流体类比中，θ 扮演流体能量的角色，π_a 是热流，$-\mu$ 是压强，σ_{ab} 是粘滞应力张量。由此可见，不仅角对称群的表示理论能导出二维流体类比，类光曲面上的引力动力学也同样如此，这与膜范式 [104] 一致。非膨胀视界的概念 [67,119] 现在可以用这套现代语言简单理解为能量 θ 和耗散张量 σ_{ab} 都为零的理想流体。

In order to connect these results to the LQG literature, we now translate the analysis of dynamics on a null surface in terms of tetrads and connection variables.

为了将这些结果与圈量子引力文献联系起来, 我们现在用标架和联络变量转换对类光曲面上动力学的分析。

This will allow us to understand the role of the Immirzi parameter and show that a non-zero Immirzi parameter implies a discretization of the spectra of the area operator which is the corner charge of internal boost symmetry. We also show that the spinor variables are naturally quantized as Fock space variables. These results are the null slice representation of the results derived in sections ”Representation Theory of Corner Symmetries” and ”Classical Theory and Symplectic Structure.”

这将让我们理解伊米尔齐参数的作用，并证明非零伊米尔齐参数意味着面积算符的谱是离散的——面积算符是内部 boost 对称的角荷。我们还证明旋量变量自然地按福克空间变量量子化。这些结果是“角对称的表示理论”和“经典理论与辛结构”两节推导结论的类光切片表示。

Boundary Frame Fields and Boundary Lagrangian

边界标架场与边界拉格朗日量

Loop gravity is based on the Einstein-Cartan geometry. Just like we have triads and tetrads, we can also have orthonormal co-frames intrinsic to the null boundary. The intrinsic metric is positive semi-definite and has one degenerate null direction. We introduce co-dyads m_a and $\bar{m}_a \in \Omega^1(\mathcal{N}:\mathbb{C})$ that diagonalize the signature $(0++)$ metric of the null surface

圈引力基于爱因斯坦-嘉当几何。正如我们拥有三分量标架和四分量标架，我们也可以得到类光边界内蕴的标准正交余标架。类光边界的内蕴度量是半正定的，拥有一个退化零方向。我们引入余二元体 m_a 和 $\bar{m}_a \in \Omega^1(\mathcal{N}:\mathbb{C})$，它们可将类光曲面的符号差 $(0++)$ 度量对角化

$$q_{ab} = 2m_{(a)\bar{m}_b}. \tag{43}$$

Given q_{ab} , the co-dyad $(m_a, \bar{m}_a)$ is unique up to a local $\mathrm{U}(1)$ transformation sending m_a into $\mathrm{e}^{\mathrm{i}\varphi}m_a$. This transformation will play an important role below. Besides the metric q_{ab} , there is also a canonical 2-form at the boundary, namely the area element

给定 q_{ab} ，余二元体 $(m_a, \bar{m}_a)$ 在局部 $\mathrm{U}(1)$ 变换下唯一，该变换将 m_a 映射为 $\mathrm{e}^{\mathrm{i}\varphi}m_a$ 。这个变换会在下文起到重要作用。除度量 q_{ab} 外，边界上还存在一个正则 2-形式，即面积元

$$\varepsilon_{ab} = -2\mathrm{i}m_{[a}\bar{m}_{b]}. \tag{44}$$

The oriented area of any two-dimensional cross section S of $\mathcal{N}$ is then given by the integral $\mathrm{Ar}[S] = -\mathrm{i}\int_S m \wedge \bar{m}$.

任意二维截面 S 相对于 $\mathcal{N}$ 的定向面积可由积分 $\mathrm{Ar}[S] = -\mathrm{i}\int_S m \wedge \bar{m}$ 给出。

The loop representation is based on quantum states that carry half-integer spin labels. Rather than working with Lorentz vector-valued p -forms, it is therefore more natural to work with a spinor representation of the frame bundle. Using the soldering forms $\sigma^{AA'}{}_I$ between Lorentz vectors and spinors, we map Lorentz vectors into pairs of spinors,

圈表示基于携带半整数自旋标记的量子态。因此，相较于处理洛伦兹向量值 p 形式，使用标架丛的旋量表示更为自然。借助洛伦兹向量与旋量之间的焊接形式 $\sigma^{AA'}{}_I$ ，我们将洛伦兹向量映射为旋量对，

$$V^{AA'} = \frac{\mathrm{i}}{\sqrt{2}}\sigma^{AA'}{}_I V^I,\ V^I = \frac{\mathrm{i}}{\sqrt{2}}\bar{\sigma}_{AA'}{}^I V^{AA'}. \tag{45}$$

Under this map, the Plebański 2-form $E^{IJ} = e^I \wedge e^J$ and the $\mathrm{SO}(1,3)$ connection ω^{IJ} split into left-handed and right-handed parts. We can now move to the construction of the boundary Lagrangian.

在该映射下，普雷班斯基 2-形式 $E^{IJ} = e^I \wedge e^J$ 与 SO(1,3) 联络 ω^{IJ} 分解为左手部分与右手部分。现在我们可以开始构造边界拉格朗日量了。

In the spinor representation, the action neatly splits into left-handed and right-handed parts,

在旋量表示下，作用量恰好分解为左手部分与右手部分，

$$S_M[A, e] = \left[\frac{\mathrm{i}}{8\pi\gamma G}(\gamma + \mathrm{i})\int_M \sum_{AB} \wedge F^{AB}\right] + \mathrm{cc}. \tag{46}$$

We now need to explain how to couple this action to a boundary that is null. First of all we note that the pullback of the self-dual 2-form $\sum_{AB}$ onto the null surface

现在我们需要说明如何将该作用量与类光边界耦合。首先我们注意到自对偶 2-形式 $\sum_{AB}$ 在类光曲面上的拉回

$$\varphi^*_{\mathcal{N}}\sum_{AB} = e_{(A} \wedge \bar{m}\ell_{B)}. \tag{47}$$

These boundary fields have a neat geometric interpretation. The spinor ℓ_A is the square root of the null generators of $\mathcal{N}$. If $e_{AA'}$ is the bulk tetrad, we have, in fact,

这些边界场具有清晰的几何解释: 旋量 ℓ_A 是 $\mathcal{N}$ 类光母线的平方根。若 $e_{AA'}$ 是体四分量标架，实际上我们有:

$$\varphi^*_{\mathcal{N}}\ell_a = 0,\ \ell_a = \mathrm{i}e_{AA'a}\ell^A\overline{\ell}^{A'}. \tag{48}$$

The 2-form $e_A \wedge \bar{m}$, on the other hand, encodes the rest of the intrinsic geometry of $\mathcal{N}$. If we contract, for example, $e_A \wedge \bar{m}$ with ℓ^A , we obtain the area 2-form,

另一方面，2-形式 $e_A \wedge \bar{m}$ 编码了 $\mathcal{N}$ 其余的内蕴几何。例如，若将 $e_A \wedge \bar{m}$ 与 ℓ^A 缩并，我们就能得到面积 2-形式，

$$\varepsilon = \mathrm{i}e_A \wedge \bar{m}\ell^A. \tag{49}$$

It is possible to show, see [100], that one can recover the entire intrinsic geometry of the null surface from $\eta_A = e_A \wedge \bar{m}$ and ℓ^A alone.

可以证明 (参见文献 [100])，仅从 $\eta_A = e_A \wedge \bar{m}$ 和 ℓ^A 就可以还原出类光曲面的全部内蕴几何。

To introduce a basis in the spin bundle, we introduce a dual spinor k_A , whose square returns the Ehresmann connection introduced earlier in section ”Carrollian Fluid Conservation Law,” i.e.,

为了给旋量丛引入一组基，我们引入对偶旋量 k_A ，其平方给出前文“开尔文流体守恒定律”小节中引入的埃雷斯曼联络，即:

$$k_a = -\mathrm{i}e^{AA'}{}_a k_A \bar{k}_{A'}, \ \text{ such that } \ k_A \ell^A = 1. \tag{50}$$

In terms of this basis, the spinor-valued 2-form $e_A \wedge \bar{m}$ admits the decomposition

旋量值 2-形式 $e_A \wedge \bar{m}$ 可以在这组基下分解为

$$e_A \wedge \bar{m} = -(\ell_A k + k_A m) \wedge \bar{m}, \tag{51}$$

where k is the Ehresmann connection on the null boundary (now viewed as a 1-form intrinsic to $\mathcal{N}$). The curvature of this connection determines the Carrollian acceleration φ and the Carrollian vorticity w [112, 114] .

其中 k 是类光边界上的埃雷斯曼联络 (此处视为内蕴于 $\mathcal{N}$ 的 1-形式)。该联络的曲率决定了开尔文加速度 φ 和开尔文涡量 w [112, 114] 。

$$\mathrm{d}k = -\varphi k \wedge \bar{m} - \overline{\varphi} k \wedge m + \mathrm{i}w m \wedge \bar{m}. \tag{52}$$

The exterior derivative of the $\mathrm{U}(1)$ dyad m_a , on the other hand, determines a $\mathrm{U}(1)$ connection Γ and the shear σ and expansion θ of the null boundary via

另一方面，$\mathrm{U}(1)$ 二重矢量 m_a 的外导数通过如下方式确定 $\mathrm{U}(1)$ 联络 Γ ，以及类光边界的剪切 σ 和膨胀 θ

$$\mathrm{d}m = -\mathrm{i}\Gamma \wedge m + \frac{1}{2}\theta k \wedge m + \sigma k \wedge \bar{m}. \tag{53}$$

Taking into account the Frobenius theorem, we thus see that for non-vanishing shear, there is an obstruction to find a potential for m_a . If there is shear, we cannot find a holomorphic coordinate $z : \mathcal{N} \to \mathbb{C}$ that would satisfy $m \propto \mathrm{d}z$.

结合弗罗贝尼乌斯定理可知，对于非零剪切，存在阻碍我们寻找 m_a 势的障碍。若存在剪切，我们无法找到满足 $m \propto \mathrm{d}z$ 的全纯坐标 $z : \mathcal{N} \to \mathbb{C}$ 。

Derivatives of the boundary spinors are important as well. Without assuming special boundary conditions, the only available derivative is the pullback of the $\mathrm{SL}(2, \mathbb{C})$ covariant (exterior) derivative from the bulk. We call this derivative $D = \varphi^*_{\mathcal{N}} \nabla$ and it provides a Carrollian connection as explained in the previous section. Given the spinor basis (k^A, ℓ^A) at the null boundary, the spin coefficients are

边界旋量的导数同样重要。在不假设特殊边界条件的情况下，唯一可用的导数是从体空间拉回的 $\mathrm{SL}(2, \mathbb{C})$ 协变 (外) 导数。我们将该导数称为 $D = \varphi^*_{\mathcal{N}} \nabla$ ，如上一节所述，它给出了一个卡罗尔联络。给定类光边界上的旋量基 (k^A, ℓ^A) ，旋系数为

$$k_A D \ell^A = \frac{1}{2\mathrm{i}} (\Gamma + \mathrm{i}\omega), \tag{54}$$

$$\ell_A D \ell^A = -\left(\frac{1}{2}\theta m + \sigma \bar{m}\right), \tag{55}$$

$$k_A D k^A = -\frac{1}{2}\left(\theta_{(k)} - \mathrm{i}w\right)\bar{m} - \overline{\sigma}_{(k)}\bar{m} + \left(\overline{\varphi} + \overline{\pi}\right)k, \tag{56}$$

where the 1-form $\omega \in \Omega^1(\mathcal{N} : \mathbb{R})$ encodes the non-affinity κ of the null generators ℓ^a as well as the momentum aspect π_a of the Carrollian fluid, i.e.,

其中 1-形式 $\omega \in \Omega^1(\mathcal{N} : \mathbb{R})$ 编码了类光生成元 ℓ^a 的非仿性 κ，以及卡罗尔流体的动量方面 π_a，即

$$\omega = \kappa k + \overline{\pi} m + \pi \bar{m}. \tag{57}$$

The 1-form $\omega \in T^*\mathcal{N}$ determines extrinsic data - it cannot be obtained from exterior derivatives of m and k intrinsic to $\mathcal{N}$. In the same way, the transversal expansion $\overline{\theta} = q^{ab}\nabla_a k_b$ and the transversal shear $\sigma_{(k)} = \bar{m}^a\bar{m}^b\nabla_a k_b$ are extrinsic quantities on $\mathcal{N}$ (Here, we extended the boundary fields $\ell^a \in T\mathcal{N}$ and $(k_a, m_a, \bar{m}_a) \in T^*\mathcal{N}$ into a Newman-Penrose null tetrad $(k^a, \ell^a, m^a, \bar{m}^a)$ in a neighborhood of $\mathcal{N}$ such that $q^{ab} = m^a\bar{m}^a +$ cc.). In the following, it will be useful to combine Γ and ω into a single complexified boundary connection $A \in \Omega^1(\mathcal{N} : \mathbb{C})$, which is given by

1-形式 $\omega \in T^*\mathcal{N}$ 确定了外禀数据——它无法从内蕴于 $\mathcal{N}$ 的 m 和 k 的外导数得到。同理，横向膨胀 $\overline{\theta} = q^{ab}\nabla_a k_b$ 和横向剪切 $\sigma_{(k)} = \bar{m}^a\bar{m}^b\nabla_a k_b$ 是 $\mathcal{N}$ 上的外禀量 (此处我们将边界场 $\ell^a \in T\mathcal{N}$ 和 $(k_a, m_a, \bar{m}_a) \in T^*\mathcal{N}$ 延拓到 $\mathcal{N}$ 邻域内的纽曼-彭罗斯类光四矢 $(k^a, \ell^a, m^a, \bar{m}^a)$，使得 $q^{ab} = m^a\bar{m}^a +$ cc.)。在下文中，将 Γ 和 ω 组合为单个复化边界联络 $A \in \Omega^1(\mathcal{N} : \mathbb{C})$ 会带来便利，该联络由下式给出

$$A = -\mathrm{i}\Gamma + \omega - \theta k. \tag{58}$$

The boundary fields carry a representation of the boundary symmetries. First of all, they are clearly covariant under internal $\mathrm{SL}(2,\mathbb{C})$ gauge transformations and dif-feomorphisms that preserve $\mathcal{N}$. This is trivially so. Then they also admit a complex-ified $\mathrm{U}(1)$ symmetry that acts on boundary fields $\left(\ell^A, A, \sigma\right) \to \left(\mathrm{e}^{\frac{1}{2}(\lambda+i\phi)}\ell^A, A + \ d(\lambda + i\phi), e^{\lambda+2i\phi}\sigma\right)$, where ϕ is a $\mathrm{U}(1)$ gauge parameter intrinsic to $\mathcal{N}$ and λ generates internal boosts.

边界场承载边界对称性的表示。首先，它们显然在保持 $\mathcal{N}$ 的内部 $\mathrm{SL}(2,\mathbb{C})$ 规范变换和微分同胚下协变，这是显然的。其次，它们也允许作用在边界场 $\left(\ell^A, A, \sigma\right) \to \left(\mathrm{e}^{\frac{1}{2}(\lambda+i\phi)}\ell^A, A + \ d(\lambda + i\phi), e^{\lambda+2i\phi}\sigma\right)$ 上的复化 $\mathrm{U}(1)$ 对称性，其中 ϕ 是内蕴于 $\mathcal{N}$ 的 $\mathrm{U}(1)$ 规范参数，λ 生成内部 boost 变换。

At the null boundary $\mathcal{N}$, we now have to choose specific boundary conditions. At the kinematical level, the bulk fields $e_{AA'}$ and $A^A{}_B$ are completely arbitrary. What is kept fixed is a gauge equivalence class of boundary fields,

在类空边界 $\mathcal{N}$ 上，我们现在必须选定特定边界条件。在运动学层面，体场 $e_{AA'}$ 和 $A^A{}_B$ 完全任意，保持固定的是边界场的规范等价类,

$$g = [m, A, \theta] / \sim, \ \delta g = 0. \tag{59}$$

The equivalence relations that define this equivalence class are diffeomorphisms of the null boundary, complexified U(1) transformations, in addition to the following shift symmetries and conformal rescalings of the boundary data:

定义该等价类的等价关系包括类空边界的微分同胚、复化 U(1) 变换，此外还有以下边界数据的平移对称性和共形重标度:

$$[m, A] \sim \left[m, A + \frac{\mathrm{i}}{2}\frac{\alpha}{\gamma + \mathrm{i}}\right], \sim \left[m, A + \zeta\bar{m} + \bar{\zeta}m\right], \sim \left[\mathrm{e}^{f} m, A\right], \tag{60a}$$

where the 1-form α and the function f are real, but ζ is complex. It is easy to check that this equivalence class is determined by two local degrees of freedom along $\mathcal{N}$, i.e., the two local degrees of freedom of gravitational radiation along a null boundary.

其中 1 形式 α 和函数 f 为实量，ζ 为复量。不难验证，该等价类由 $\mathcal{N}$ 方向的两个局域自由度确定，即类空边界上引力辐射的两个局域自由度。

Given the boundary conditions (60), the corresponding boundary action [72] is

给定边界条件 (60)，对应的边界作用量 [72] 为

$$S_{\mathcal{N}} = \left[\frac{\mathrm{i}}{8\pi\gamma G}(\gamma + \mathrm{i})\int_{\mathcal{N}}\left(e_A \wedge \bar{m} \wedge \left(D - \frac{1}{2}A\right)\ell^A - \frac{\theta}{4}e_A \wedge e^A \wedge \bar{m}\right)\right] + \mathrm{cc.}$$

(61)

This action has a simple structure. The first term is just a gauged kinetic term for a configuration variable ℓ^A , which is charged under $\mathrm{SL}(2, \mathbb{C}) \times \mathrm{U}_{\mathbb{C}}(1)$, and $e_A \wedge \bar{m}$ is the corresponding chiral momentum variable. The second term is a simple quadratic Hamiltonian. The coupled bulk plus boundary action is the sum of the two terms

该作用量结构简单: 第一项是构型变量 ℓ^A 的规范动能项，ℓ^A 带 $\mathrm{SL}(2, \mathbb{C}) \times \mathrm{U}_{\mathbb{C}}(1)$ 下的荷，$e_A \wedge \bar{m}$ 是对应的手征动量变量；第二项是简单的二次哈密顿量。耦合的体加边界作用量为两项之和

$$S_{\text{bulk+boundary}} = S_{\mathcal{M}} + S_{\mathcal{N}} . \tag{62}$$

Notice that the shift symmetry (60a) depends on the Barbero-Immirzi parameter γ . In fact, variations of the boundary action with respect to the boundary 1-form $\alpha \in \Omega^1(\mathcal{N} : \mathbb{R})$ impose a constraint, namely the reality condition

注意平移对称性 (60a) 依赖于巴贝罗-伊米里齐参数 γ 。实际上，边界作用量对边界 1 形式 $\alpha \in \Omega^1(\mathcal{N} : \mathbb{R})$ 的变分会给出一个约束，即实性条件

$$\varepsilon = \bar{\varepsilon}. \tag{63}$$

From the variation of the boundary spinors e_A and ℓ^A , we obtain additional boundary field equations that propagate the boundary spinors along $\mathcal{N}$. The resulting boundary field equations can be written in the following compact form:

通过对边界旋量 e_A 和 ℓ^A 变分，我们得到额外的边界场方程，用来沿 $\mathcal{N}$ 演化边界旋量。得到的边界场方程可以写成如下紧致形式:

$$\bar{m} \wedge \left(D - \frac{1}{2}A\right) \ell^A = \frac{\theta}{2} e^A \wedge \bar{m}, \tag{64}$$

$$\left(D + \frac{1}{2}A\right) \wedge (e_A \wedge \bar{m}) = 0. \tag{65}$$

If we take into account the decomposition of the spin connection with respect to the spin frame $\left(k^A, \ell^A\right)$, it is easy to check that these boundary field equations are satisfied on any embedded null surface. This is important to note, because it means that the boundary field equations impose no constraints on the free data along $\mathcal{N}$. In other words, the boundary field equations (64) and (65) can be satisfied on any three-dimensional null boundary.

如果我们考虑自旋联络相对于自旋标架 $\left(k^A, \ell^A\right)$ 的分解，不难验证这些边界场方程在任意嵌入的类空面上都成立。这一点很重要，因为它意味着边界场方程不会对 $\mathcal{N}$ 上的自由数据施加任何约束。换句话说，边界场方程 (64) 和 (65) 可以在任意三维类空边界上成立。

Given the boundary conditions and boundary field equations, we infer the symplectic potential. On a partial Cauchy surface M that intersects $\mathcal{N}$ in a cross section S , there are a contribution from the bulk and an additional boundary contribution

给定边界条件和边界场方程，我们可以推导出辛势。在与 $\mathcal{N}$ 相交于截面 S 的部分柯西曲面 M 上，存在体贡献和额外的边界贡献

$$\Theta_{\Sigma} = \frac{\mathrm{i}}{8\pi\gamma G}(\gamma + \mathrm{i}) \left[\int_{\Sigma} \Sigma_{AB} \wedge \delta A^{AB} - \oint_{\partial M} e_A \wedge \bar{m} \delta \ell^A\right] + \mathrm{cc.} \tag{66}$$

The expression for the symplectic structure on the null surface simplifies if we restrict ourselves to only those variations $\delta[\cdot]$ that preserve the ruling of the null surface, i.e., $\delta\ell^a \sim \ell^a$, see [72]. Under this condition, we obtain

如果我们只考虑保持类空面母线结构的变分 $\delta[\cdot]$ (即满足 $\delta\ell^a \sim \ell^a$ ，参见文献 [72])，类空面上辛结构的表达式会简化。在该条件下我们得到

$$\Theta_{\mathcal{N}} = -\frac{1}{8\pi\gamma G} \int_{\mathcal{N}} \varepsilon \wedge \delta A^{(\gamma)} - \frac{\mathrm{i}}{8\pi\gamma G} \int_{\mathcal{N}} \left[(\gamma + \mathrm{i})\, \ell_A D \ell^A \wedge \delta(k \wedge \bar{m}) - \mathrm{cc.}\right],$$

(67)

where $k_a \in \Omega^1(\mathcal{N} : \mathbb{R})$ is the Ehresmann connection such that $k_a \ell^a = 1$. Notice that the expression for the symplectic potential has the familiar structure from the loop gravity holonomy flux algebra, with fluxes $\varepsilon, k \wedge m$, and $k \wedge \bar{m}$ that are dual to 1-forms or connections. In fact, the area 2-form ε is dual to the $\mathrm{U}(1)$ connection

其中 $k_a \in \Omega^1(\mathcal{N} : \mathbb{R})$ 是满足 $k_a \ell^a = 1$ 的埃雷斯曼联络。注意辛势的表达式具有圈引力全通流代数中常见的结构，其中通量为 $\varepsilon, k \wedge m$ ， $k \wedge \bar{m}$ 是 1 形式或联络的对偶。实际上，面积 2 形式 ε 是 $\mathrm{U}(1)$ 联络的对偶

$$A^{(\gamma)} = \Gamma + \gamma\omega \tag{68}$$

The connection $A^{(\gamma)}$ is an abelian version of the SU(2) Ashtekar-Barbero connection, namely the sum of the intrinsic $\mathrm{U}(1)$ connection Γ and the extrinsic curvature ω . In fact, given the $\mathrm{U}(1)$ gauge transformations, it is easy to check that $A^{(\gamma)}$ transforms as an abelian $\mathrm{U}(1)$ connection. If, on the other hand, we rescale $A^{(\gamma)}$ by γ^{-1} , we obtain an abelian connection for the internal boost symmetry $\ell^a \to \mathrm{e}^{\lambda}\ell^a$. Let us now discuss two important limiting cases.

> 该联络 $A^{(\gamma)}$ 是 SU(2) 阿西特卡-巴贝罗联络的阿贝尔版本，即内蕴 $\mathrm{U}(1)$ 联络 Γ 与外曲率 ω 之和。事实上，给定 $\mathrm{U}(1)$ 规范变换后，很容易验证 $A^{(\gamma)}$ 按照阿贝尔 $\mathrm{U}(1)$ 联络变换。另一方面，如果我们用 γ^{-1} 对 $A^{(\gamma)}$ 做标度变换，就能得到内部 boost 对称性 $\ell^a \to \mathrm{e}^{\lambda}\ell^a$ 的阿贝尔联络。下面我们来讨论两个重要的极限情况。

In the case of an isolated horizon $\mathcal{N} \simeq \mathbb{S}^2 \times \mathbb{R}$, we have no gravitational radiation crossing the null boundary. Shear and expansion of the null generators ℓ^a vanish. Therefore, the second term in the expression for the pre-symplectic potential (67) disappears, and we are left with the first term. Since the area element ε_{ab} is conserved, the entire geometry becomes effectively two-dimensional. Given an arbitrary two-dimensional cross section $S \simeq \mathbb{S}^2$ of $\mathcal{N}$, the symplectic potential is now simply given by

> 在孤立视界 $\mathcal{N} \simeq \mathbb{S}^2 \times \mathbb{R}$ 的情况下，没有引力辐射穿过类光边界。类光生成元 ℓ^a 的切变和膨胀均为零。因此，前辛势 (67) 表达式中的第二项消失，只剩下第一项。由于面积元 ε_{ab} 守恒，整个几何有效化为二维。任取 $\mathcal{N}$ 的一个二维截面 $S \simeq \mathbb{S}^2$ ，辛势现在可简单写为

$$\Theta_{\mathrm{IH}} = \frac{1}{8\pi\gamma G}\oint_S \varepsilon h^{-1}\delta h, \tag{69}$$

where $h(x) = \mathrm{Pexp}\left(-i\int_{\gamma(\mathbf{x})} A^{(\gamma)}\right)$ is the $\mathrm{U}(1)$ holonomy along the null generator based at $\mathbf{x}$. This defines the phase space $[T^*\mathrm{SU}(1)]^S$. Notice that the area 2-form is the generator of $\mathrm{U}(1)$ gauge transformations, and hence the area is quantized.

> 其中 $h(x) = \mathrm{Pexp}\left(-i\int_{\gamma(\mathbf{x})} A^{(\gamma)}\right)$ 是基于 $\mathbf{x}$ 的类光生成元上的 $\mathrm{U}(1)$ 和乐。这就定义了相空间 $[T^*\mathrm{SU}(1)]^S$ 。注意到面积 2-形式是 $\mathrm{U}(1)$ 规范变换的生成元，因此面积是量子化的。

Another important limiting case is when we send $\mathcal{N}$ to null infinity $\mathcal{I}^{\pm}$. In this case, the symplectic potential (67) returns the usual radiative phase space [94].

> 另一个重要的极限情况是我们将 $\mathcal{N}$ 移至类光无穷远 $\mathcal{I}^{\pm}$ 。此时，辛势 (67) 回到通常的辐射相空间 [94]。

Spinor Representation

旋量表示

From the previous analysis, we see that at the corner of null surface, where the $\sum$ intersects the null surface, we obtain the Poisson brackets

通过之前的分析我们看到，在 $\sum$ 与类光面相交的类光面角区，我们得到泊松括号

$$\left\{\widetilde{\pi}_A(\mathbf{x}), \ell^B(\mathbf{y})\right\}_S = \delta_A^B \tilde{\delta}_S(\mathbf{x},\mathbf{y}), \left\{\widetilde{\overline{\pi}}_{A'}(\mathbf{x}), \overline{\ell}^{B'}(\mathbf{y})\right\}_S = \delta_{A'}^{B'} \tilde{\delta}_S(\mathbf{x},\mathbf{y}), \tag{70}$$

where we introduced the momentum spinor $\widetilde{\pi}_A$ as the pullback of the boundary field $e_A \wedge \bar{m}$ to the cross section S , i.e.,

其中我们将动量旋量 $\widetilde{\pi}_A$ 定义为边界场 $e_A \wedge \bar{m}$ 拉回至截面 S 的结果，即:

$$\widetilde{\pi}_A = \frac{\mathrm{i}}{8\pi\gamma G}(\gamma + \mathrm{i})\varphi_S^*(e_A \wedge \bar{m}). \tag{71}$$

A number of Dirac observables can be written in terms of these boundary fields. For example, the generator of infinitesimal tangential diffeomorphism associated with $\xi^a \in T\sum$ is given by

多个狄拉克可观测量都可以用这些边界场表示。例如，与 $\xi^a \in T\sum$ 关联的无穷小切微分同胚生成元为

$$P_\xi = \oint_S \xi^a \left[\widetilde{\pi}_A D_a \ell^A\right] + \mathrm{cc}. \tag{72}$$

In the same way, we find the generators of complexified U(1) transformations,

同理，我们可以得到复化 U(1) 变换的生成元,

$$\widetilde{\pi}_A \ell^A = \widetilde{K} + \mathrm{i}\tilde{L} = \frac{\gamma + \mathrm{i}}{8\pi\gamma G}\varepsilon, \tag{73}$$

where $\tilde{L}$ is a $U(1)$ generator, $\widetilde{K}$ is a boost generator, and ε is the area form. In terms of harmonic oscillators, $\tilde{L}$ is the difference of two number operators, and $\widetilde{K}$ is a two-mode squeeze operator, see [43]. The spectrum of $\widetilde{K}$ is continuous, and the spectrum of $\tilde{L}$ is discrete. The area 2-form (49) is the contraction of the boundary spinors. At the kinematical level, this surface density is complex. For the area to be real, we have to impose the simplicity constraint $\widetilde{K} - \gamma\tilde{L} = 0$.

其中 $\tilde{L}$ 是 $U(1)$ 生成元，$\widetilde{K}$ 是快度 boost 生成元，ε 是面积形式。用谐振子表述，$\tilde{L}$ 是两个数算符的差，$\widetilde{K}$ 是双模压缩算符，参见文献 [43]。$\widetilde{K}$ 的谱是连续的，$\tilde{L}$ 的谱是离散的。面积二形式 (49) 是边界旋量的缩并。在运动学层面，该面密度是复数值。为了让面积为实，我们需要施加简洁性约束 $\widetilde{K} - \gamma\tilde{L} = 0$ 。

When acting on any physical state, the area density will be proportional to $\tilde{L}$, which has a discrete spectrum. Hence the area is quantized. This selects a representation of the canonical commutation relations. In this way, the quantization of area on the null surface boundary agrees with our analysis in section "Representation Theory of Corner Symmetries." Both results only rely on the compactness of the orbits generated by the area density $\varepsilon \propto \tilde{L}$. If we use a spin network representation, the boundary spinors $(\widetilde{\pi}_A, \ell^A)$ will be excited at a number of punctures. We could also use, however, a more standard Fock representation [43], where $\widetilde{\pi}_A$ and ℓ^A are continuous. In either case, the result is the same. To summarize the Barbero-Immirzi parameter deforms the boundary symmetries in such a way that the area spectrum becomes discrete.

作用在任意物理态上时，面积密度正比于 $\tilde{L}$ ，而 $\tilde{L}$ 具有离散谱，因此面积是量子化的。这选出了正则对易关系的一种表示。由此，类光面边界上面积的量子化与我们在“角对称性表示论”一节中的分析一致。两个结果都仅依赖于面积密度 $\varepsilon \propto \tilde{L}$ 生成轨道的紧致性。如果使用自旋网表示，边界旋量 $(\tilde{\pi}_A, \ell^A)$ 会在若干刺点处激发。但我们也可以采用更标准的福克表示 [43]，其中 $\tilde{\pi}_A$ 和 ℓ^A 都是连续的。无论哪种情况，结论都相同: 综上，巴贝罗-伊米爾齐参数对边界对称性的形变使得面积谱成为离散谱。

Summary and Perspectives

总结与展望

In this chapter, we have discussed three main ideas and related results. The first result is about the classical phase space of general relativity in the presence of inner boundaries of a space-like or null slice. When there is a boundary, an otherwise unphysical gauge redundancy in the bulk turns into a physical degree of freedom intrinsic to the boundary. The corresponding conjugate elements on phase space are the canonical generators of the boundary symmetries. It is for this reason that the generators of diffeomorphisms turn into surface charges that measure quasi-local observables of the gravitational field. These observables generalize the 10 Minkowskian Noether charges for the Poincaré group into an infinite set of boundary charges that satisfy the so-called corner symmetry algebra. We then reviewed a number of past results from loop quantum gravity, where the boundary conditions and resulting boundary symmetries have led to important results, from local entanglement [44] to the entropy counting of black hole horizons [45] and to renormalization and the continuum limit of the theory [49]. Then, we explained one of the key results of the corner symmetry program thus far, namely the fact that the loop quantization of area can be understood from the quantization of the boundary modes alone [35, 43, 72] . Since there is a unique area element on a null surface, the construction becomes particularly transparent for such boundaries. The area 2-form at a two-dimensional cross section turns into a surface charge, whose spectrum is discrete. From the perspective of the gravitational phase space in a finite region, this surface charge is a complete Dirac observable. The discreteness of the spectrum of the corresponding quantum operator can be traced back to the presence of the Barbero-Immirzi parameter in the action. The Barbero-Immirzi parameter does not affect the commutation relations for the radiative modes in the bulk, but it deforms the algebra of boundary symmetries and their generators. This discreteness has been proven to follow from representing the corner symmetry as in (20). Finally, we have also reviewed a number of results on how the Einstein equations induce a field theory for a Carrollian fluid on the null surface boundary and how this can also be described naturally within the tetrad formulation used in loop quantum gravity. This furthermore connects seamlessly with the description of isolated horizons developed in the LQG black hole entropy counting.

在本章中，我们讨论了三个核心观点及相关结论。第一个结论围绕存在类空或类零切片内边界时广义相对论的经典相空间展开。当存在边界时，体中原本非物理的规范冗余会转变为边界本身固有的物理自由度。相空间中对应的共轭量就是边界对称性的正则生成元。正因如此，微分同胚生成元转变为测量引力场准局域可观测量的面荷。这些可观测量将庞加莱群的 10 个闵可夫斯基诺特荷推广为满足所谓转角对称代数的无穷多边界荷。接着我们回顾了圈量子引力的诸多过往研究成果，其中边界条件与由此产生的边界对称性已经得到了诸多重要结论，从局域纠缠 [44] 到黑洞视界的熵计数 [45]，再到理论的重整化与连续极限 [49]。随后我们阐释了转角对称纲领迄今为止的核心结论之一，即面积的圈量子化可以仅通过边界模的量子化得到理解 [35, 43, 72] 。由于类零面上存在唯一的面积元，该构造对此类边界而言格外清晰明了。二维截面上的面积 2-形式转变为面荷，其谱是离散的。从有限区域引力相空间的角度来看，这个面荷是一个完备的狄拉克可观测量。对应量子算符谱的离散性可以追溯到作用量中存在的巴贝罗-伊米里齐参数。该参数不影响体中辐射模的对易关系，但会改变边界对称性及其生成元的代数。已经证明，这种离散性是 (20) 中表示转角对称的必然结果。最后，我们还回顾了诸多相关结论，即爱因斯坦方程如何在类零面边界上诱导出卡罗尔流体的场论，以及这一过程如何能在圈量子引力使用的四标架表述中自然描述。此外，这还能与 LQG 黑洞熵计数中发展出的孤立视界描述无缝衔接。

Some developments were not covered in our review. For example, there is a growing body of research in quantum foundations and quantum information science on quantum reference frames and their relation with edge modes [120- 123]. Quantum reference frames are dual (in the phase space sense of the word) to the generators of coordinate transformations. Recent results by Brukner and collaborators gave an operational definition of quantum reference frames, see, e.g., [124]. This definition includes a prescription for how to jump between quantum reference frames and identify properties of a quantum state that remain invariant under such transformations. It turns out, for example, that quantum entanglement is an observer-dependent notion and depends on the frame of reference. Strengthening further the connection between corner symmetry and quantum reference frames is a promising area of development.

本篇综述并未涵盖部分研究进展。例如，量子基础与量子信息科学领域中，关于量子参考系及其与边界模关系的研究正不断增多 [120-123]。量子参考系与坐标变换生成元是 (相空间意义上的) 对偶关系。布鲁克纳及其合作者的最新研究给出了量子参考系的操作化定义，参见例如 [124]。该定义包含了如何在不同量子参考系间切换、识别在这类变换下保持不变的量子态性质的方法。例如已有结论表明，量子纠缠是依赖于观测者的概念，且与参考系有关。进一步加强转角对称与量子参考系的关联是一个颇具前景的发展方向。

Altogether, the idea we arrive at is that of local holography. While AdS holography on asymptotic time-like boundaries is made possible by reflecting boundary conditions, which prevent from including radiation [125], celestial-Carrollian Holography allows the inclusion of radiation and is adapted to the description of physically realistic setups [19-21,126-128]. Instead of considering null or time-like boundaries at infinity, local holography aims at obtaining constraints on quantum gravity from the local corner symmetry structure associated with any arbitrary subregion defined by an entangling cut. This idea should not necessarily be viewed as a new proposal for quantum gravity, but rather as a consistency framework that brings a universal bottom-up perspective for all candidate approaches. In this review, we have discussed the lessons from and for loop quantum gravity, but it will be interesting and important to discuss local holography in other contexts as well.

综上，我们最终得到的观点就是局域全息。渐近类时边界上的 AdS 全息依靠反射边界条件实现，这会排除辐射 [125]，而天球-卡罗尔全息则允许包含辐射，适用于描述物理现实的场景 [19-21,126-128]。局域全息并不考虑无穷远处的类零或类时边界，其目标是从任意由纠缠割定义子区域关联的局域转角对称结构出发，得到量子引力的约束。这一观点不必被看作量子引力的全新提案，更像是一个自洽框架，为所有候选量子引力方法提供了普适的自下而上视角。本篇综述我们讨论了来自圈量子引力以及面向圈量子引力的启示，但在其他语境下讨论局域全息也同样会是有趣且重要的工作。

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